**HIGH DENSITY MATTER**

## 1. Problem 2

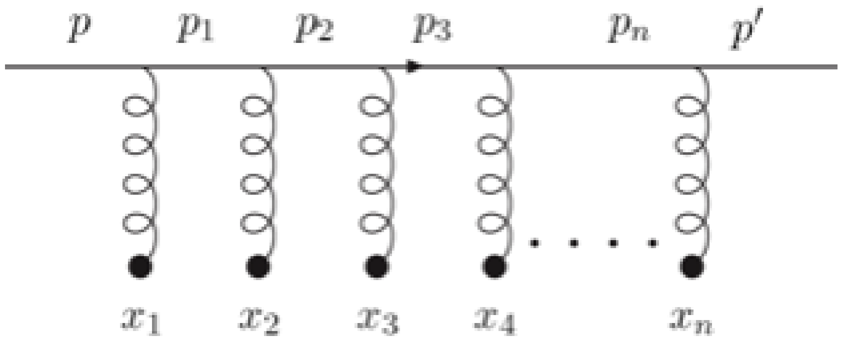


Figure 1. example caption

Consider a massless, fast-moving quark moving propagating through a "dense" medium (it could be a nucleus or a QGP) along the z (3rd) direction. It is convenient to work in light-cone coordinates:

$$a^\mu = (a^+, a^-, a_\perp), \quad \text{with} \quad a^\pm = \frac{a^0 \pm a^3}{\sqrt{2}}\,;\; a_\perp \equiv (a^1, a^2) \tag{1}$$

In these coordinates the scalar product is written as

$$a \cdot b = a^+ b^- + a^- b^+ + a_\perp \cdot b_\perp. \tag{2}$$

Thus, the momentum of the incoming quark can be written as $p^\mu \simeq (p^+, 0, 0_\perp)$.

*Question 1*: How does the metric look like in these coordinates? How do you raise and lower indeces?

The quark will interact with the color sources in the medium through multiple gluon exchange. The contribution to the scattering matrix from a single scattering is

$$S_1(p,p') = \int d^4x\, e^{i(p-p')\cdot x} \bar{u}^{s'}(p')\, igA^a_\mu\, t^a\, \gamma^\mu\, u^s(p), \tag{3}$$

where $p'$ is the outgoing quark momentum.

*Question 2*: Using the *eikonal* approximation, $p \sim p'$ (why?), and the relation $\frac{1}{2}\sum_{spins} \bar{u}^{s'}(p)\gamma^\mu u^s(p) = 2p^\mu$, show that $S_1$ can be writen as:

$$S_1(p,p') \simeq 2\pi\delta(p'^+ - p^+)2p^+ \int dx_\perp e^{ix_\perp\cdot(p'_\perp - p_\perp)} \left[ig \int dx^+ A^-(x^+, x_\perp)\right], \tag{4}$$

where $A^\mu \equiv A^\mu t^a$ henforth. Relying again in the eikonal approximation, $p^+ >>$, demonstrate that the contribution from two rescatterings is given by

$$S_2(p',p) \simeq 2\pi\delta(p'^+ - p^+)2p^+ \int dx_\perp e^{-ix_\perp\cdot(p'_\perp - p_\perp)} \frac{1}{2}\mathcal{P}\left[ig \int dx^+ A^-(x^-, x^+)\right]^2 \tag{5}$$

Finally, use the expression above to guess the contribution from n rescatterings, $S_n(p',p)$. Then, demonstrate that the total scattering matrix is given by:

$$S(p',p) = \sum_{n=0}^{\infty} S_n(p',p) \simeq 2\pi\delta(p'^+ - p^+)2p^+ \int dx_\perp e^{-ix_\perp\cdot(p'_\perp - p_\perp)} W(x_\perp), \tag{6}$$

where $W(x_\perp$ is the following Wilson line:

$$W(x_\perp) = \mathcal{P}\exp\left[ig \int dx^+ A^-(x^-, x_\perp)\right]. \tag{7}$$