Track fitting and alignment
(in ATLAS)

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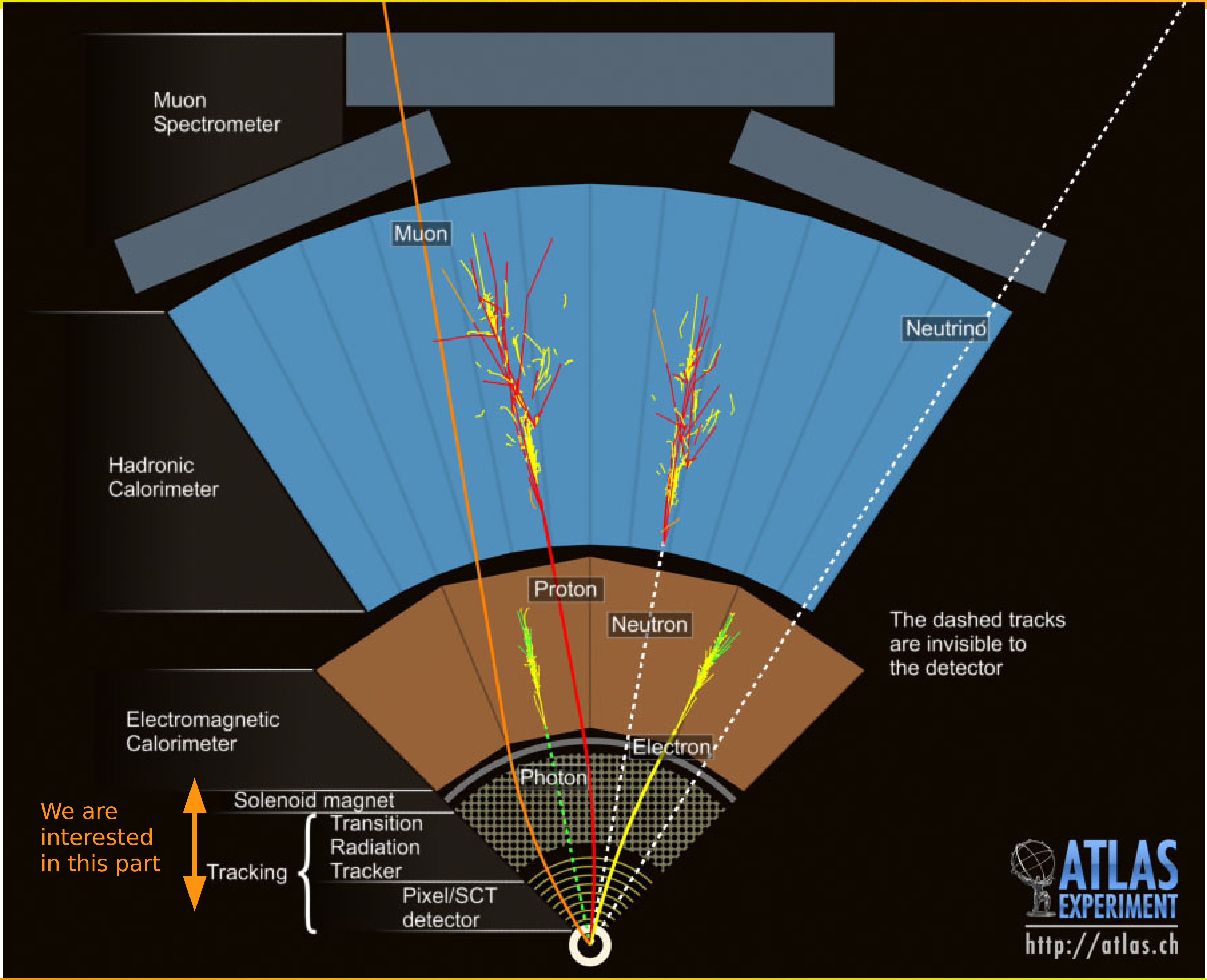
Outline

- Track fitting
  - Basic ideas & concepts
  - Basic formulae
  - Pattern recognition
  - Track fitting with χ^2 and Kalman filter techniques
  - Multiple Coulomb Scattering
- Alignment
  - Basic ideas & concepts
  - Basic formulae
  - Alignment strategy
  - Alignment systematics

Disclaimer: the geometry description is an important issue that is not treated in this lecture

Particles and detectors

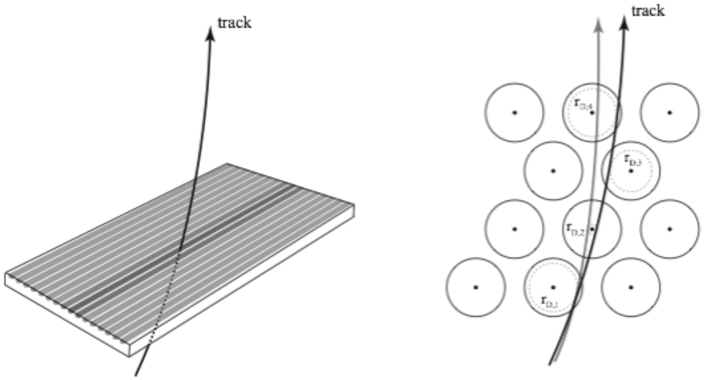


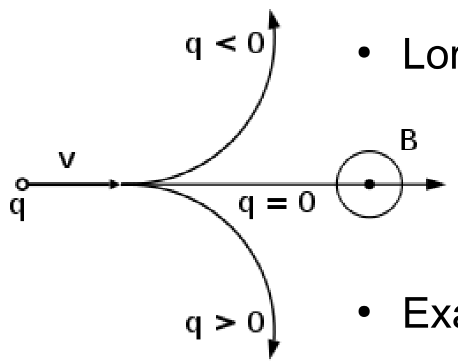
We are
interested
in this part

Introduction

- A nice performance of the Track Fitting is a key ingredient of the success of the physics program of the HEP experiments
  - An accurate determination of the charged particles properties is necessary
    - Invariant masses had to be determined with optimal precision and well estimated errors
    - Secondary vertices must be fully reconstructed: evaluate short lifetimes
    - Kink reconstruction
- Challenges for the tracking systems of the LHC detectors
  - High multiplicity of charged particles (up to 100 for $\mathscr{L} \sim 10^{34} cm^{-1} s^{-1}$)
  - Momenta of particles in the final state ranging from MeV to TeV
  - Large background from secondary activities of the particles
  - Multiple Coulomb Scattering in detector frames, supports, cables, pipes...
  - Complex modular tracking systems combining different detecting techniques, different resolutions
  - Resolutions that vary as a function of the momentum (p), azimuthal angle (ϕ), polar angle (θ) or pseudorapidity (η)
  - Very high event rates leading to large amount of data
    - with demanding requirements CPU and storage

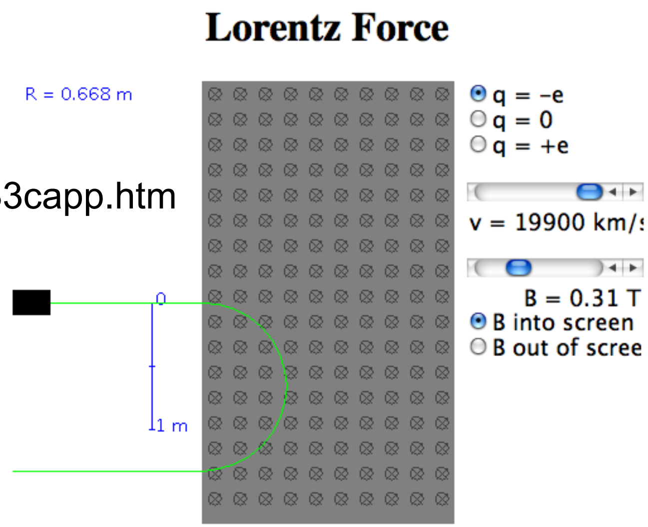
- Basic ingredients of the tracking system
  - Charged particles (+ve or -ve)
    - |q| = 1, 2
    - e,μ,π,k,p,α,...
  - Ionization detector
    - Continuous (e.g.: gas detectors)
    - Discrete (e.g.: silicon planar detectors)
  - Magnetic field (no strictly necessary)
    - Necessary if momentum determination is required
      - Some times experiments runs with magnets switched off
    - Lorentz force





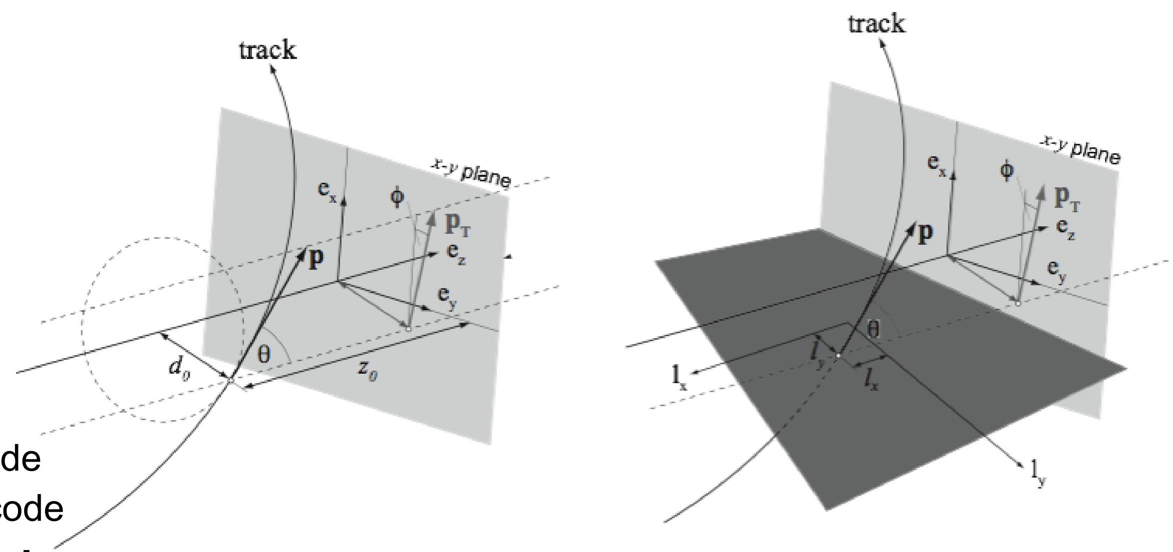
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- Example: Nice Java applet
  - http://www.lon-capa.org/~mmp/kap21/cd533capp.htm



Track parameters

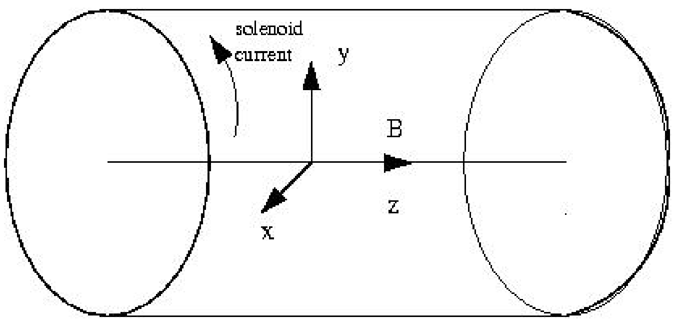
- The track can be parametrized by 5 parameters at the track perigee
  - d_0, z_0, φ_0, θ_0, p, (q)
  - d_0, z_0, φ_0, $\cot\theta_0$, p_T, (q)
- The track extrapolation to detector surfaces or elements usually requires a different parametrization
  - x_i, y_i, φ_i, θ_i, p_T, (q)
    - At intersection
  - Track extrapolation
    - From point to point
    - Active volumes
    - Passive volumes
    - Heavily used in
      - Tracking code
      - Alignment code
  - Error matrix propagation !

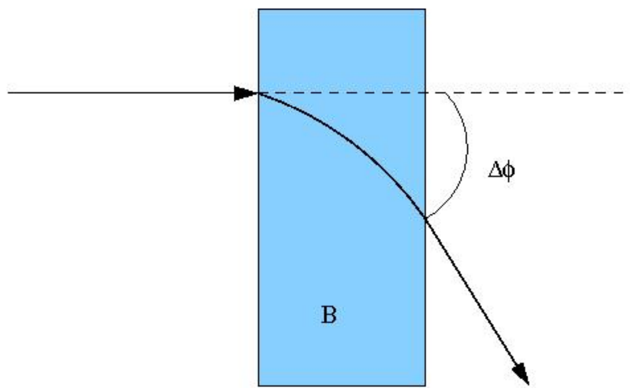


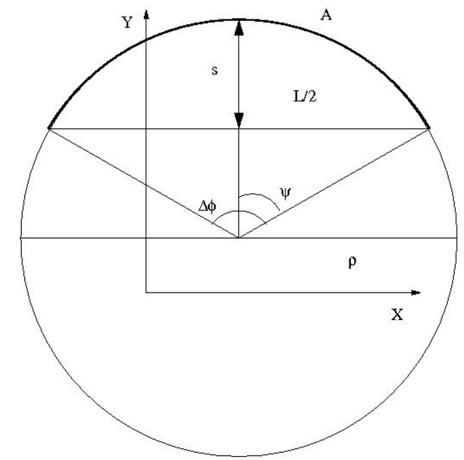
- Optimization
  - Track parameters given in the local reference frame

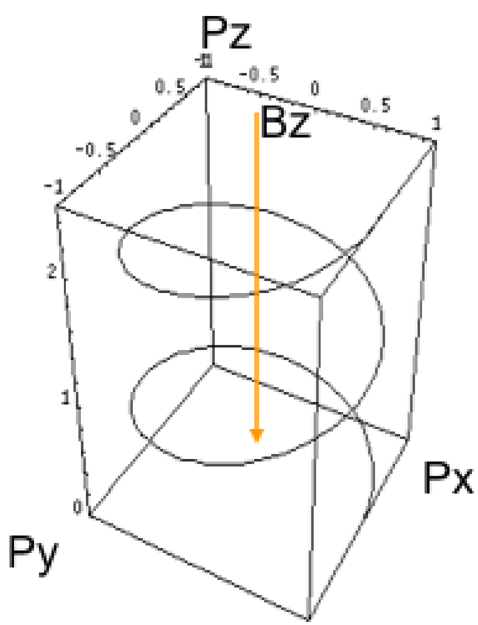
Basic track formulæ

- Consider axial (along Z) and uniform B field
  - From a solenoid field as in most of the HEP experiments trackers.
  - Charged particles follow a helicoidal path
    - Describe circles in the XY (transverse plane) due to Lorentz force
    - Move uniformly along Z









$$\vec{F} = q\vec{v} \times \vec{B}$$

$$p_T(GeV/c) = 0.3\, q\, B(T)\, \rho(m)$$

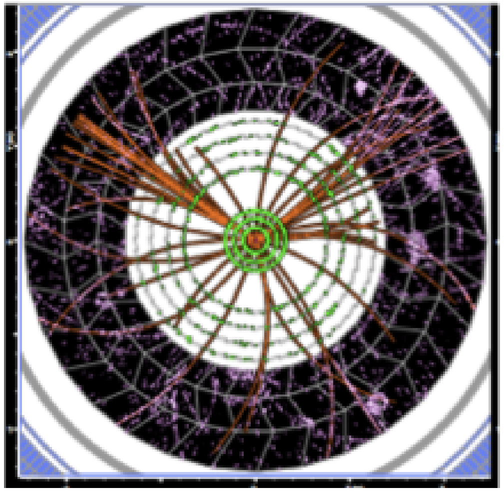
$$\rho = 0.3\, B\left(\frac{L^2}{8s} + \frac{s}{2}\right)$$

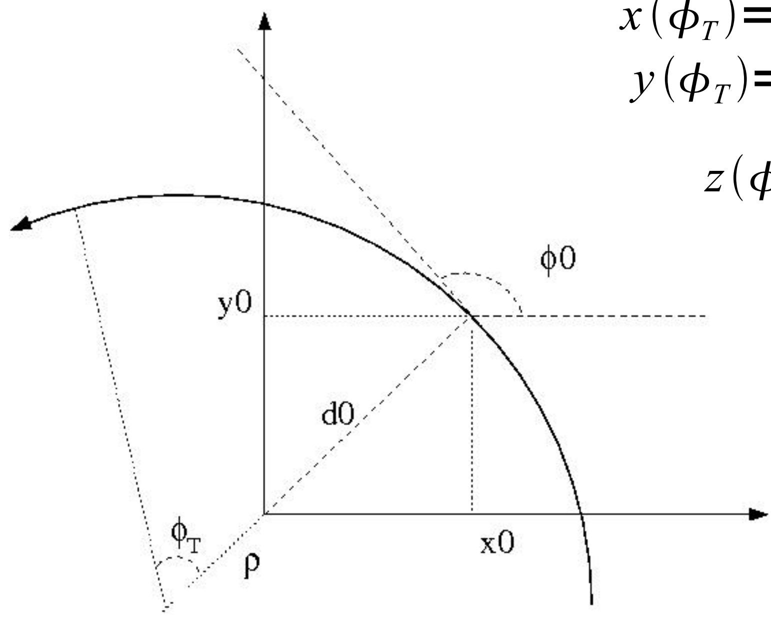
$$\frac{\delta p_T}{p_T} = \frac{\delta s}{s}$$

$$\frac{\delta p_T}{p_T} \propto \frac{\delta s}{B L^2} p_T$$

Basic track formulae

- Helix trajectory of charged particles
  - Parametrization of the helix: (x,y,z) of a trajectory point as a function of a single path parameter

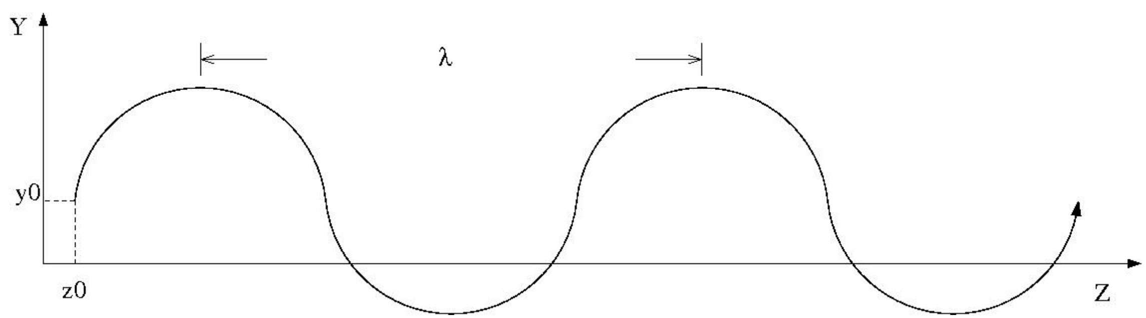




$$x(\phi_T) = -q\,\rho \sin(\phi_0 - q\,\phi_T) + (d_0 + q\,\rho)\sin\phi_0$$
$$y(\phi_T) = q\,\rho \cos(\phi_0 - q\,\phi_T) - (d_0 + q\,\rho)\cos\phi_0$$
$$z(\phi_T) = z_0 + \lambda \frac{\phi_t}{2\pi} = z_0 + (\rho \cot\theta_0)\,\phi_T$$

$$x_0 = d_0 \sin\phi_0$$
$$y_0 = -d_0 \cos\phi_0$$

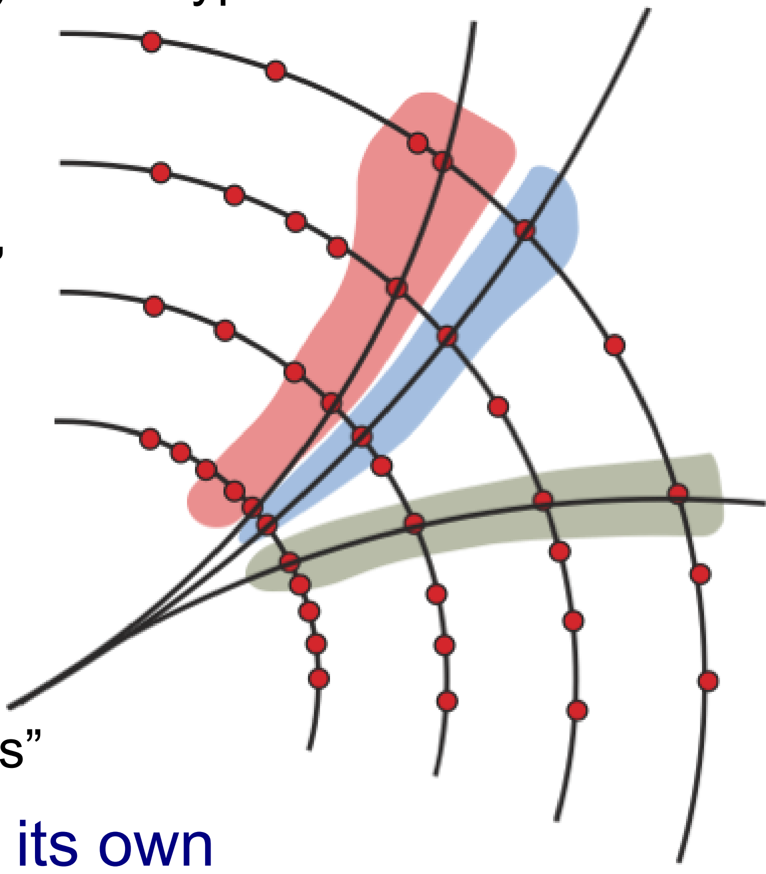
$$\rho = \frac{p_T}{0.3\,B}$$
$$p_T = p \sin\theta_0$$
$$\rho \cot\theta_0 = \frac{p}{0.3\,B} \cos\theta_0$$



See example at: http://www-jlc.kek.jp/2003oct/subg/offl/lib/docs/helix_manip/node3.html

Pattern recognition

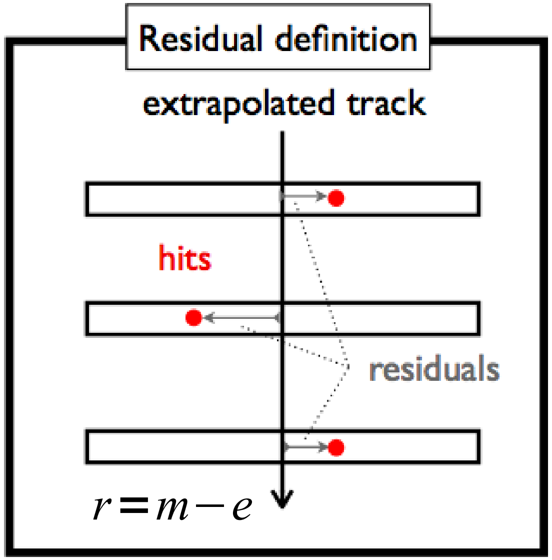
- The main goal of the pattern recognition is to associate hits to tracks
  - Efficient: all hits
  - Robust: no noise and no hits from other tracks
- Pattern recognition is a field of applied mathematics
  - It makes use of statistics, cluster analysis, combinatorial optimization, etc
  - The choice of the algorithm depends heavily in the type of measurements
    - 2D vs 3D points
  - And in the track model
    - Detector shape and B field
  - Hough space transform, template matching, minimum spanning tree, local pattern recognition



- Hit-to-track association
  - Defined by pattern recognition
  - Later altered by tracking
    - Removing bad hits & outliers
  - Noisy channels tend to be the “party spoilers”
- In summary: pattern recognition is an art on its own

Track fitting with X^2 minimization

- Use well known technique of residual minimization for track parameters determination via X^2 function
  - Usual X^2 definition
    - Residuals (r) and their errors (σ)
  - X^2 minimization w.r.t. track parameters (π)



$$X^2=\sum_{i=1}^{N_R}\left(\frac{r_i}{\sigma(r_i)}\right)^2 \qquad \frac{dX^2}{d\pi}=0 \quad \rightarrow \quad \sum_{i=1}^{N_R}\frac{r_i}{\sigma(r_i)^2}\frac{dr_i}{d\pi}=0$$

- Rewrite the X^2 using the matrix algebra:

$$r=\begin{pmatrix} r_1 \\ \vdots \\ r_{N_R}\end{pmatrix} \qquad V=\begin{pmatrix}\sigma^2(r_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma^2(r_{N_R})\end{pmatrix} \quad \rightarrow \quad X^2=r^T V^{-1} r$$

1) V may contain correlations terms as well. Therefore V is not necessarily diagonal
2) The residuals errors are taken as the intrinsic errors of the detector elements. Each hit may come from a different tracking device and has its own error

  - Apply the X^2 minimization w.r.t. track parameters (π)

$$\frac{dX^2}{d\pi}=0 \quad \rightarrow \quad \left(\frac{dr}{d\pi}\right)^T V^{-1} r=0$$

$$\pi=\begin{pmatrix}\pi_1 \\ \vdots \\ \pi_{N_T}\end{pmatrix}=\begin{pmatrix} d_0 \\ z_0 \\ \phi_0 \\ \theta_0 \\ p\end{pmatrix} \qquad \frac{dr}{d\pi}=\begin{pmatrix} dr_1/d\pi_1 & \dots & dr_1/d\pi_{N_T} \\ \vdots & \ddots & \vdots \\ dr_N/d\pi_1 & \dots & dr_N/d\pi_{N_T}\end{pmatrix}$$

Track fitting with X^2 minimization

- Taylor's expansion up to first order derivatives: $r = r(\pi_0) + \left|\frac{dr}{d\pi}\right|_{\pi_0} \delta\pi$
  - Computed at initial track parameter (π_0) estimation
  - Neglect second and higher order derivatives: $\frac{d^2 r}{d\pi_i d\pi_j} = 0$
- The minimum condition equation becomes:

$$\frac{dX^2}{d\pi} = 0 \quad \rightarrow \quad \left(\frac{dr}{d\pi}\right)^T V^{-1} r = 0 \quad \rightarrow \quad \left[\left(\frac{dr}{d\pi}\right)^T V^{-1} \left(\frac{dr}{d\pi}\right)\right] \delta\pi + \left[\left(\frac{dr}{d\pi}\right)^T V^{-1} r\right] = 0$$

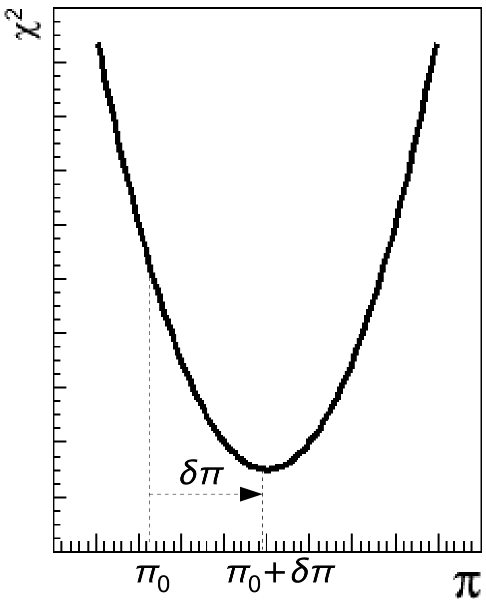
- Solving the above matrix equation requires to invert a N_T x N_T matrix

$$\delta\pi = -\left[\left(\frac{dr}{d\pi}\right)^T V^{-1} \left(\frac{dr}{d\pi}\right)\right]^{-1} \left[\left(\frac{dr}{d\pi}\right)^T V^{-1} r\right] \quad \rightarrow \quad \pi = \pi_0 + \delta\pi$$

- Pros & cons:
  - pros:
    - The inverse of the track derivatives matrix is the correlation matrix of the track parameters. So track parameters errors are computed for free :)
    - If the problem is linear then the solution is exact
  - Cons:
    - The derivatives of the residuals wrt track parameters may be hard to compute
    - If the problem is not linear then one needs to iterate

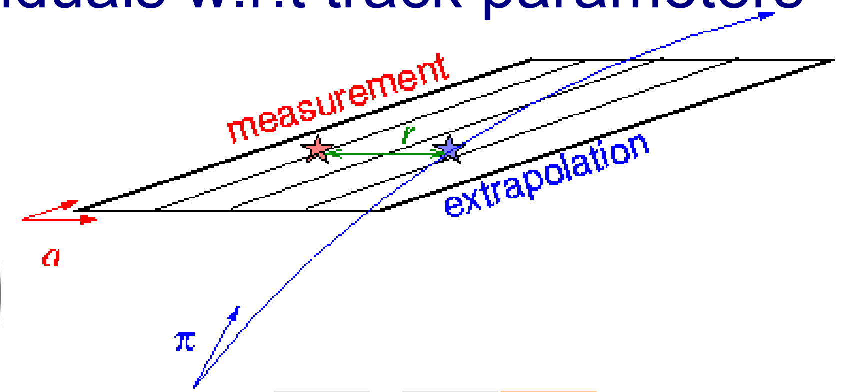
Track fitting with X² minimization

- The calculation of the derivatives of residuals w.r.t track parameters



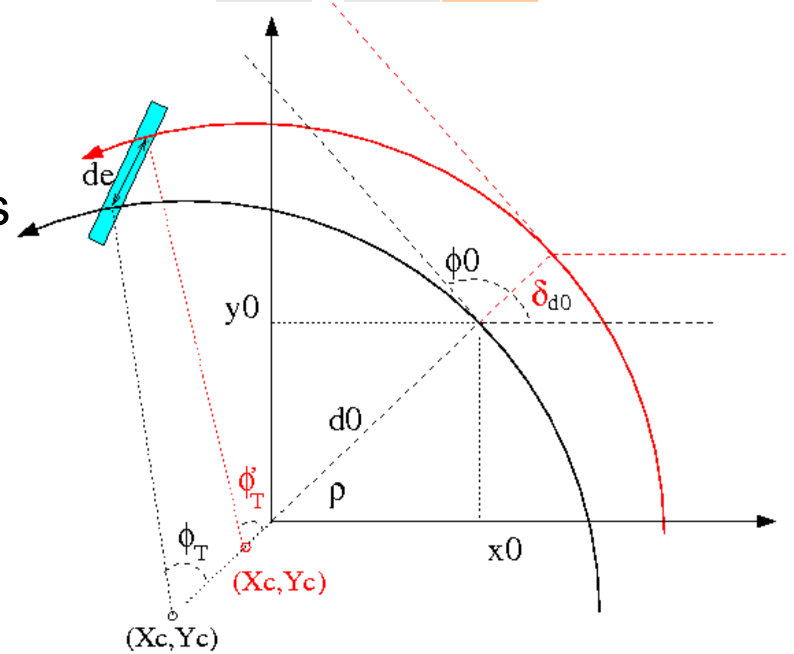
$$r = m - e \quad \rightarrow \quad \frac{dr}{d\pi} = -\frac{de}{d\pi}$$

$$m = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} \qquad e = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} = \begin{pmatrix} x(\phi_T) \\ y(\phi_T) \\ z(\phi_T) \end{pmatrix}$$



$$dx = \frac{\partial x}{\partial \pi_i} d\pi_i + \frac{\partial x}{\partial \phi_T} d\phi_T \quad \rightarrow \quad \frac{dx}{d\pi_i} = \frac{\partial x}{\partial \pi_i} + \frac{\partial x}{\partial \phi_T}\frac{d\phi_T}{d\pi_i}$$

- Intersection of the track with the detector:
  - Changes with changing track parameters
- Analytic calculations make assumptions:
  - On track model and detector conditions
    - e.g.uniform B & material description
  - Fast and reliable
- Numerical calculations
  - Time consuming, reliable & heavy use of the track extrapolation package



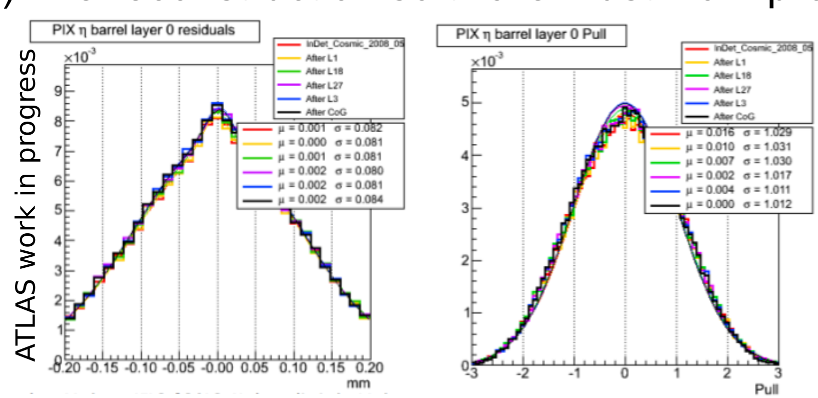
Track fitting with X^2 minimization

- Track fit with constrained track parameters
  - Beam spot, secondary vertices, invariant masses, …

$$R=\begin{pmatrix} d_0-\hat{d}_0 \\ \vdots \\ p-\hat{p} \end{pmatrix} \qquad W=\begin{pmatrix} \sigma^2(d_0) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma^2(p) \end{pmatrix} \quad \rightarrow \quad X^2=r^T V^{-1} r+R^T W^{-1} R$$

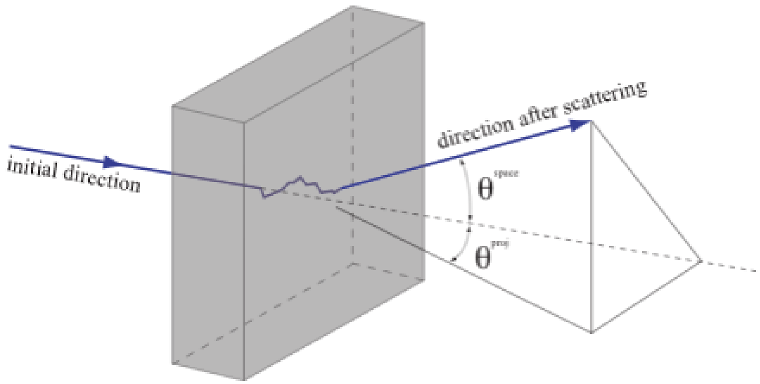
$$\delta\pi=-\left[\left(\frac{dr}{d\pi}\right)^T V^{-1}\left(\frac{dr}{d\pi}\right)+\left(\frac{dR}{d\pi}\right)^T W^{-1}\left(\frac{dR}{d\pi}\right)\right]^{-1}\left[\left(\frac{dr}{d\pi}\right)^T V^{-1} r+\left(\frac{dR}{d\pi}\right)^T W^{-1} R\right] \quad \rightarrow \quad \pi=\pi_0+\delta\pi$$

- Goodness of the fit: evaluate the pull quantities
  - When fit is correct: pulls follow a Normal distribution ($\mu=0,\sigma=1$)
  - Three conditions must be fulfilled
    1) The track model must be correct
    2) The covariance matrix of the measurement errors must be correct
    3) The reconstruction software must work properly

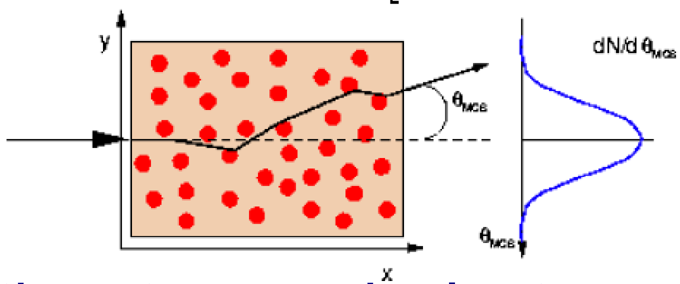


Treatment of the MCS

- The Multiple Coulomb Scattering must be included in the track fitting
  - Particle traversing material undergoes successive deflections
    - In main tracking algorithms the assumption is that the MCS angles follow a Gaussian distribution. It is know that the tails are larger than the Gaussian tails



$$\theta_{MCS}=\theta_{rms}=\frac{13.6\,MeV}{\beta c\, p} z \sqrt{\frac{x}{X_0}}\left[1+0.038\ln\left(\frac{x}{X_0}\right)\right]$$

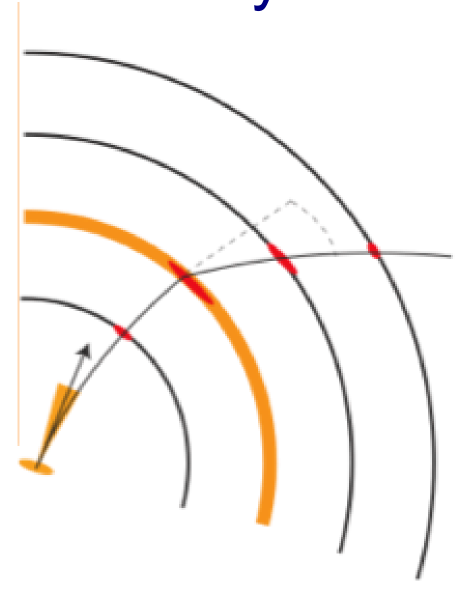


- Practical implementation in the algorithm: two equivalent ways
  - As non symmetric correlation matrix

$$V=V_{hit}+V_{MCS}=\begin{pmatrix}\sigma^2(r_1) & \dots & corr(r_1,r_{N_R})\\ \vdots & \ddots & \vdots \\ corr(r_1,r_{N_R}) & \dots & \sigma^2(r_{N_R})\end{pmatrix}$$

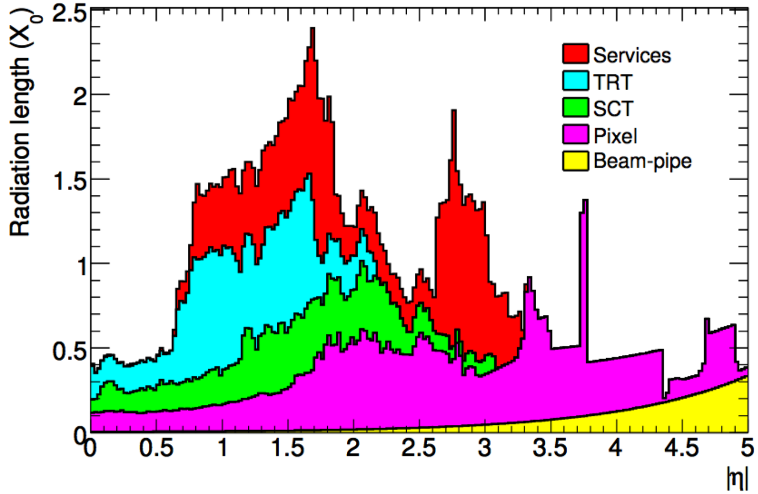
  - As extra track parameters that are fitted

$$r_\theta=\begin{pmatrix}\theta_1\\ \vdots\\ \theta_{Nscat}\end{pmatrix} \qquad \pi=\begin{pmatrix}\pi_i\\ \theta_j\end{pmatrix} \qquad X^2=r^T V^{-1} r+r_\theta V_{MCS}^{-1} r_\theta$$



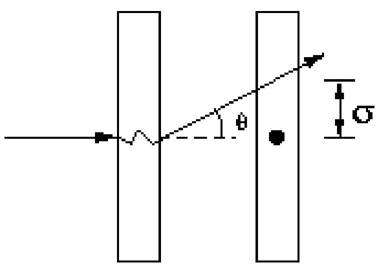
Treatment of the MCS

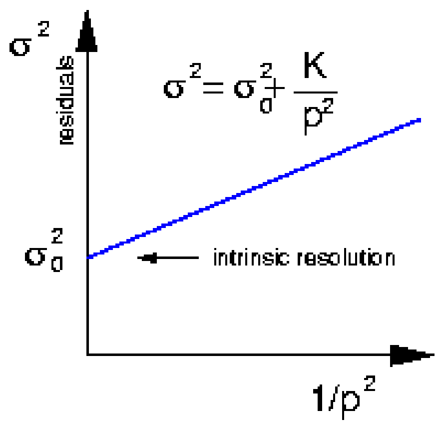
- The amount of material affects the track reconstruction

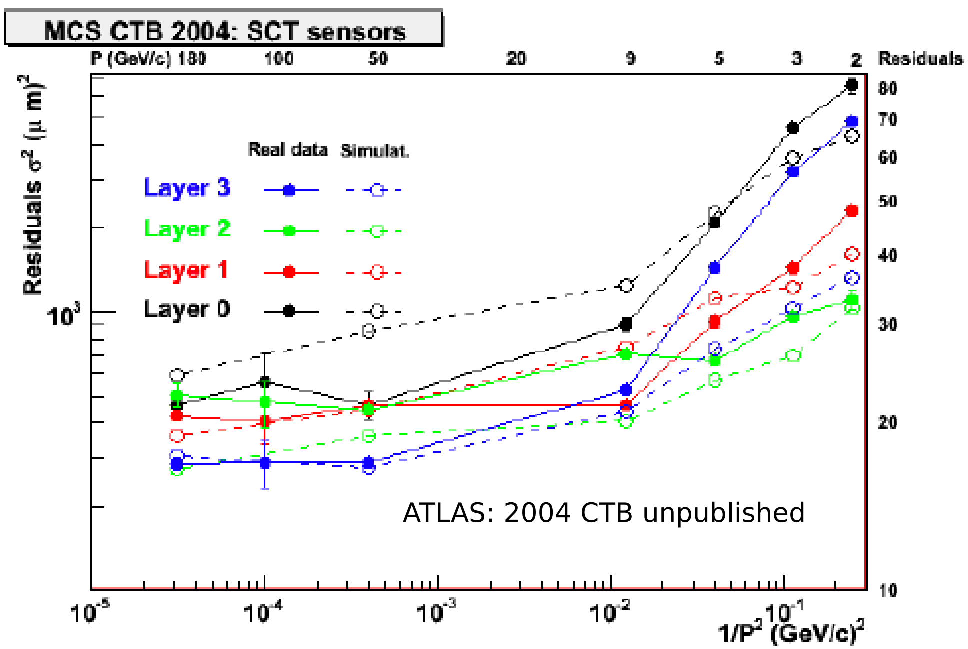


Material in the ATLAS Inner Detector expressed in units of radiation length and given as a function of the pseudorapidity

- Practical determination of the MCS and detector intrinsic resolution







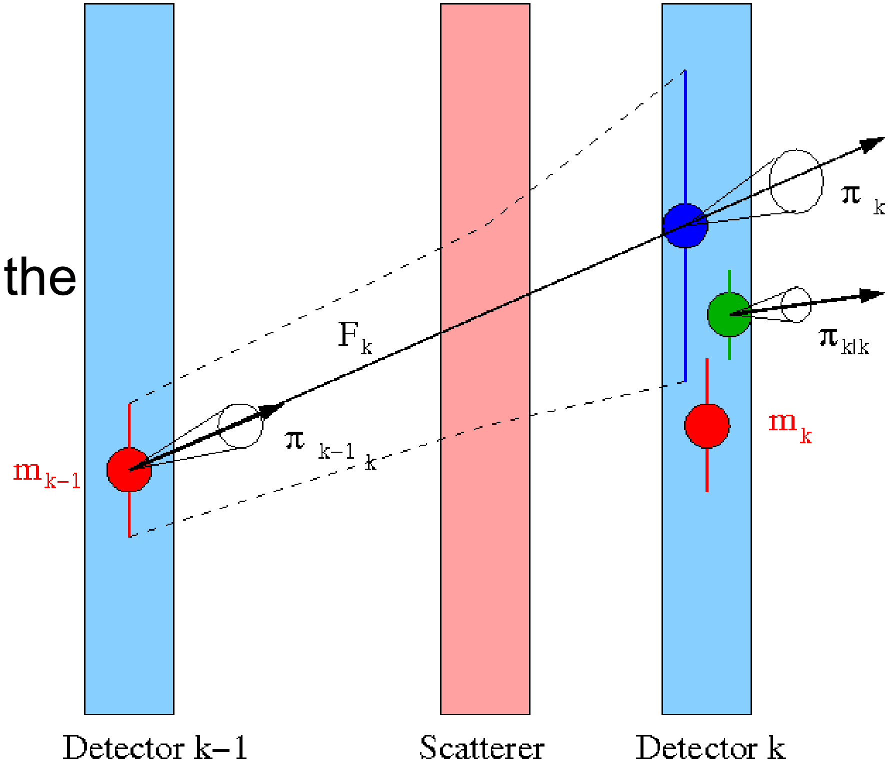
Track fitting with Kalman filter

- The Kalman filter was developed by R.E. Kalman during the 1950's
  - To solve differential matrix equations without matrix inversions
  - It is a method of estimating the states of dynamic systems
    - Soon applied to the NASA rocket trajectory control for the Apollo program
    - Military applications: compute plane trajectory by radar tracking
- Assumption:
  - The trajectory of a particle between two adjacent surfaces is described by a deterministic function plus random disturbances (material effects, etc)
  - The system equation: propagates the estate in one surface to the next

$$\pi_k = F_k(\pi_{k-1}) + P_k \boldsymbol{\delta}_k \qquad \langle \boldsymbol{\delta}_k \rangle = 0 \qquad Cov(\boldsymbol{\delta}_k) = Q_k$$

  - The measurement equation: mapping the track in the surface and considers some measurement error

$$\boldsymbol{m}_k = H_k(\boldsymbol{\pi}_k) + \varepsilon_k \qquad \langle \boldsymbol{\varepsilon}_k \rangle = 0 \qquad Cov(\boldsymbol{\varepsilon}_k) = V_k$$



Track fitting with Kalman filter

- The aim is to estimate the track parameters from the observations
  - From *j* observations and a *k*th measurement: obtain a new *k* estimate

$$\{\{\boldsymbol{m}_1,\ldots,m_j\},\quad \boldsymbol{\pi}_j\} + \boldsymbol{m}_k \rightarrow \boldsymbol{\pi}_k$$

  - **Prediction** $\pi_{k|k-1} = F_k(\pi_{k-1}) + P_k \boldsymbol{\delta}_k$
    - and its covariance matrix (error): $C_{k|k-1} = F_k C_{k-1|k-1} F_k^T + P_k Q_k P_k^T$
  - **Filtering**, based on $\pi_{k/k-1}$ and m_k:
    - It consists in minimizing the following:

$$L(\boldsymbol{\pi}_k) = (\boldsymbol{m}_k - H_k \boldsymbol{\pi}_k)^T V_k^{-1} (\boldsymbol{m}_k - H_k \boldsymbol{\pi}_k) + (\boldsymbol{\pi}_{k|k-1} - \boldsymbol{\pi}_k)^T C_{k|k-1} (\boldsymbol{\pi}_{k|k-1} - \boldsymbol{\pi}_k)$$

    - The solution should be well known by now:

$$\boldsymbol{\pi}_{k|k} = \boldsymbol{\pi}_{k|k-1} + \left[(H_k^T V^{-1} H_k) + C_{k|k-1}\right]^{-1} \left[H_k^T V^{-1} (\boldsymbol{m}_k - H_k \boldsymbol{\pi}_k)\right]$$

    - And its covariance matrix (error):

$$C_{k|k} = \left[(H_k^T V^{-1} H_k) + C_{k|k-1}\right]^{-1}$$

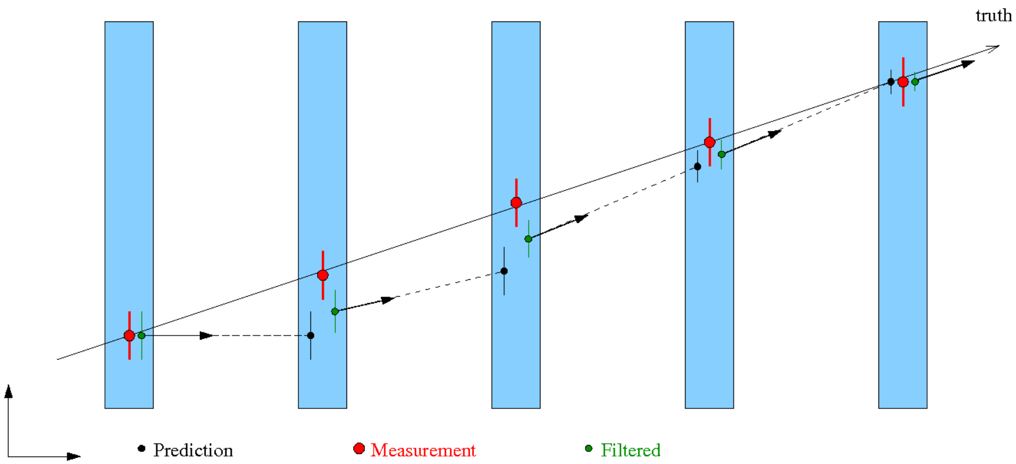
  - The residual is thus: $\boldsymbol{r}_{k|k} = \boldsymbol{m}_k - H_k \boldsymbol{\pi}_{k|k}$
    - Which allows to compute a χ^2 in order to test the goodness of the fit

$$\chi_{k|k}^2 = \boldsymbol{r}_k^T V_k^{-1} \boldsymbol{r}_k \qquad \chi^2 = \sum_k \chi_k^2$$

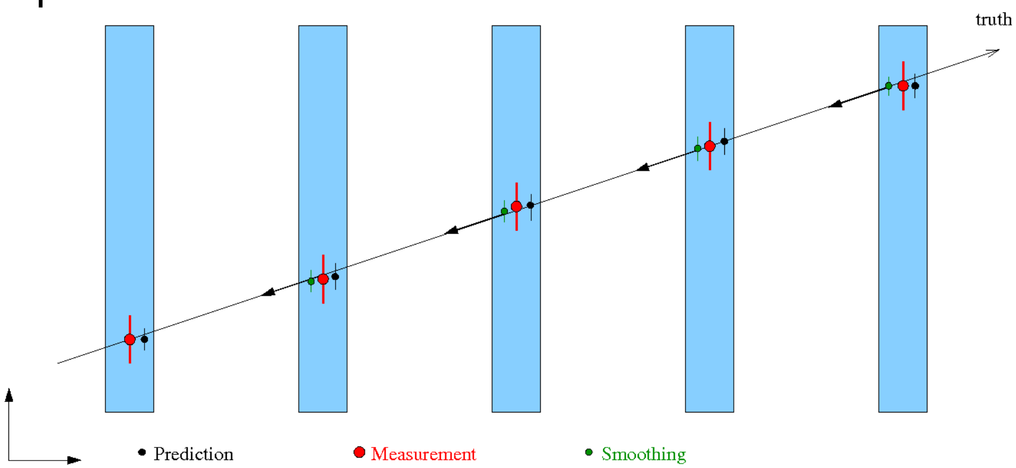
that needs some smoothing.

Track fitting with Kalman filter

- Estimate of the track parameters and state at the detector surfaces
  - Filtering from estimate *k*-1 to *k*
    - Outer points estimates have more information than inner points

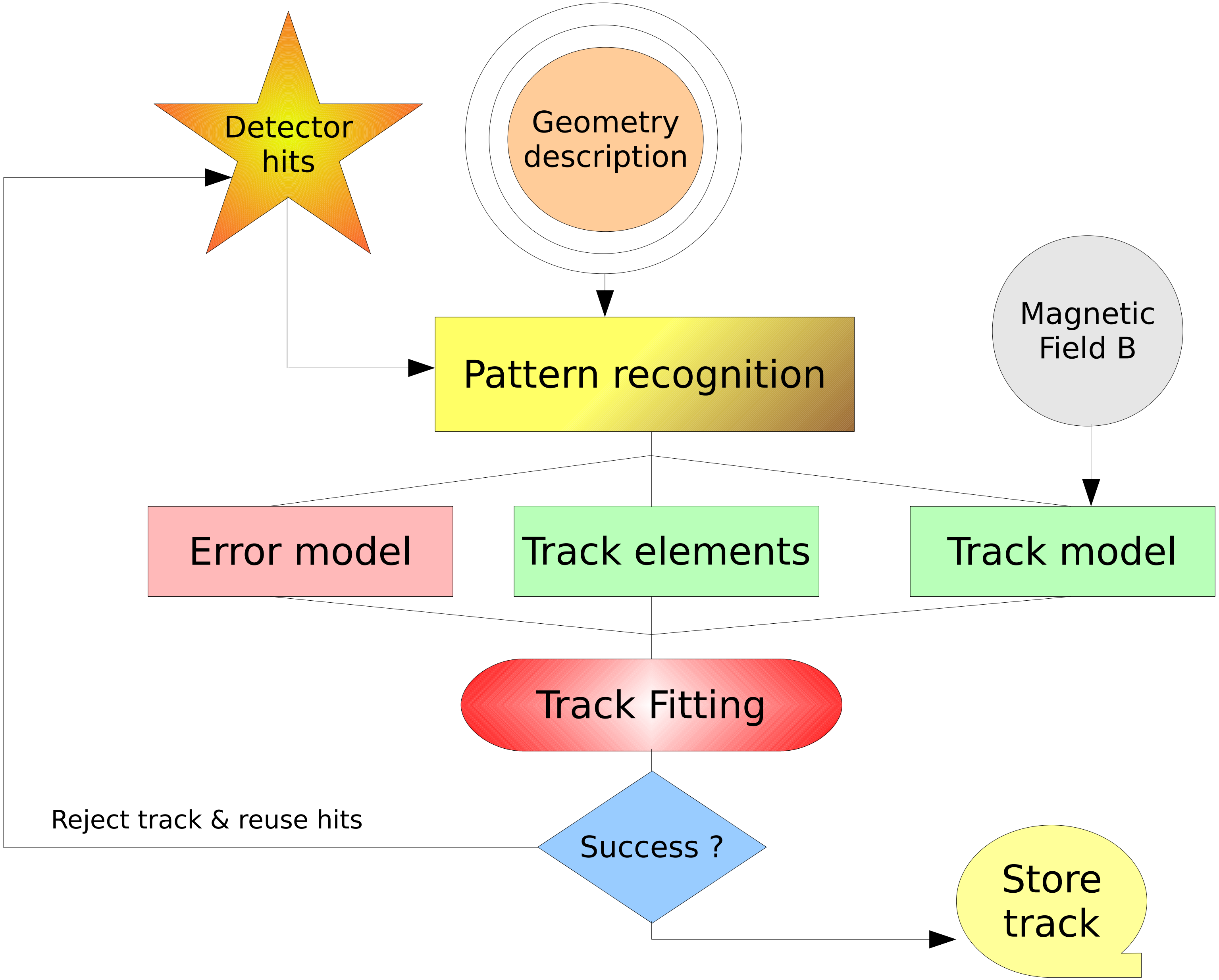


  - Smoothing: from estimate k to k-1 (sort of backward filter)
    - All points estimates have the same information



Track fitting summary

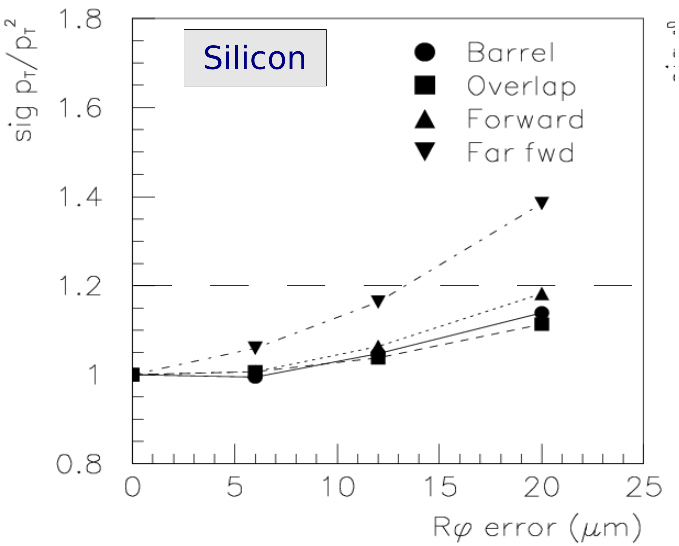
- From detector hits to particle trajectories

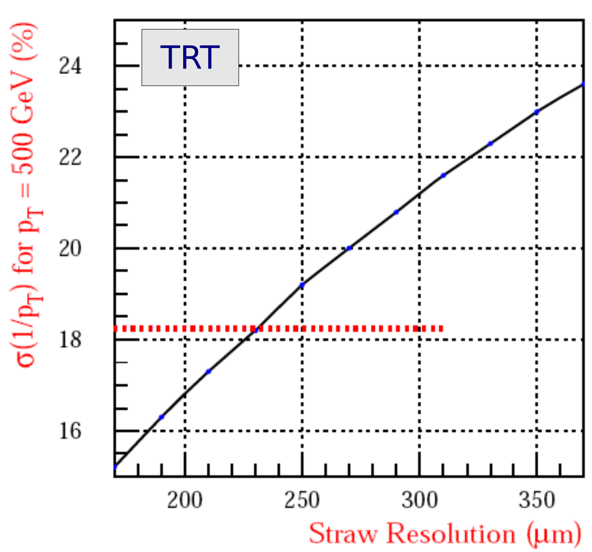


Basic ideas & concepts for alignment

- The aim of the detector alignment is to provide an accurate description of the detector geometry
  - In straight words: to know where the modules are
- The point is: the limited knowledge of the alignment constants should not lead to a significant degradation of the track parameters, beyond that of the intrinsic tracker resolution
  - In ATLAS and for the “initial physics analysis” the requirement is that the degradation should be kept below the 20%

	pixels		SCT	
	barrel	endcap	barrel	endcap
rΦ(μm)	7	7	12	12
z (μm)	20	100	50	200

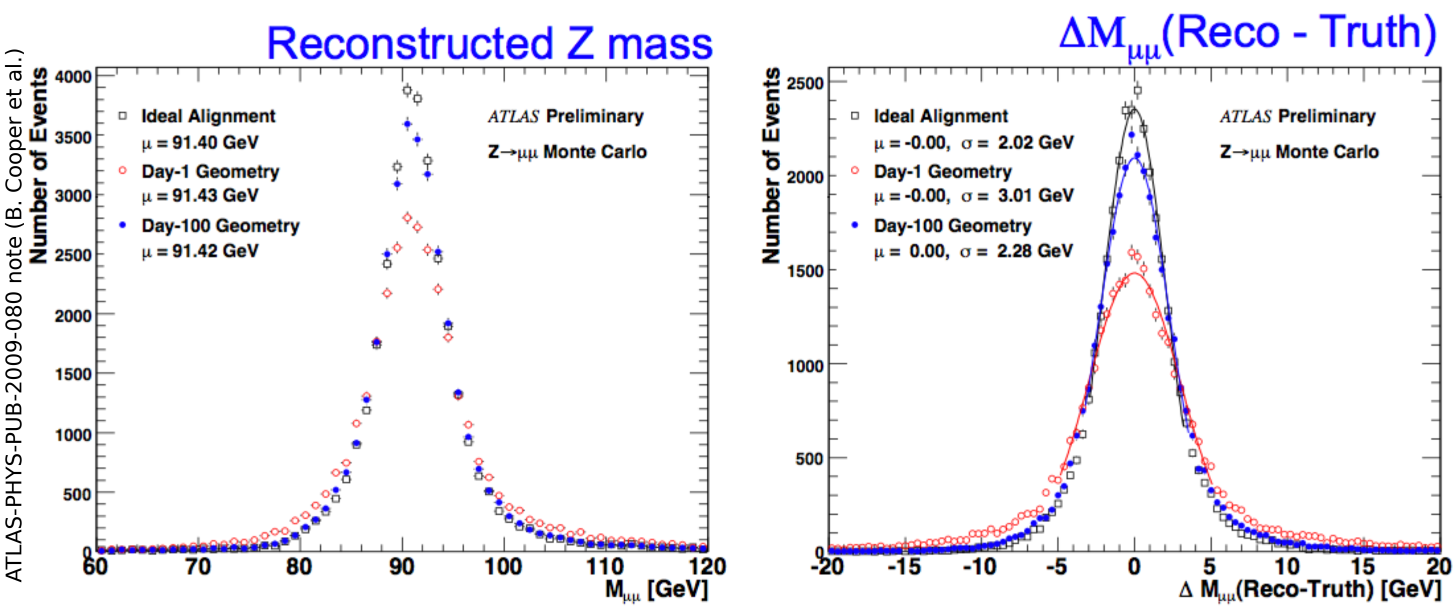




Basic ideas & concepts for alignment

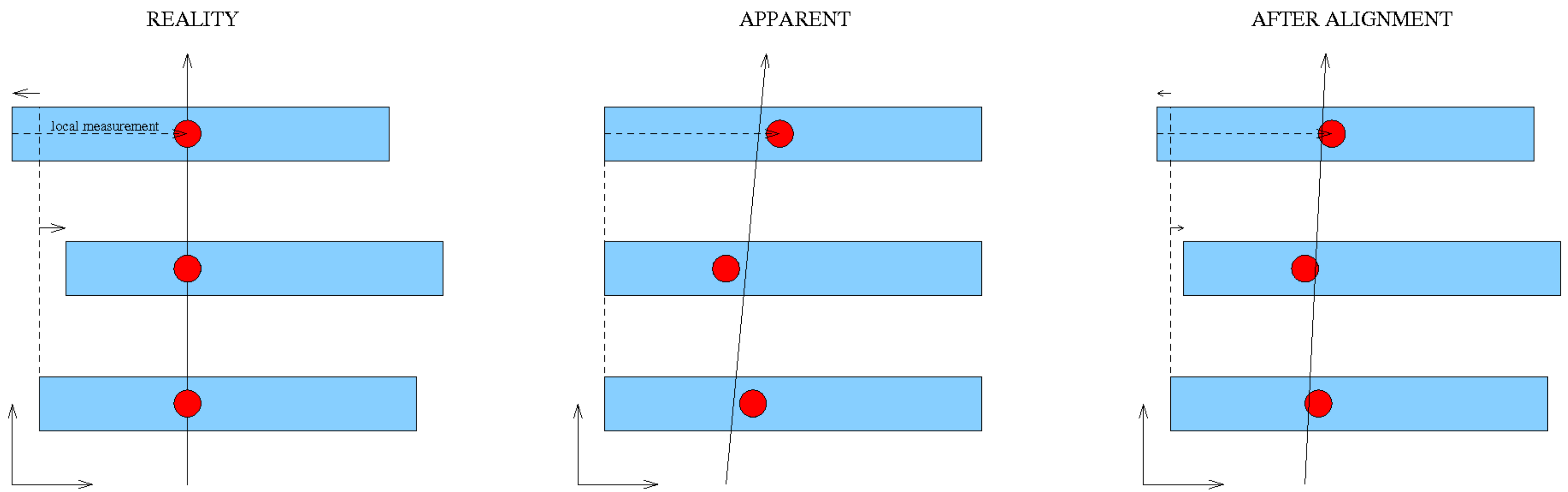
- High accuracy is required for precision measurements
  - A W-mass measurement accuracy of 15-20 MeV/c^2 requires 1μm alignment precision (S. Haywood, ATL-INDET-2000-2005)
  - Higgs mass: if 180 < m_h < 400 GeV/c^2. H→ZZ→ 4l
  - B-tagging: impact parameter & mass
- Example: Z→$\mu^+\mu^-$ analysis
  - random misalignment
  - Day-1: expected alignment accuracy for Day-1 from cosmic data
  - Day-100: estimate of situation after 100 days of collision data

	Day-1 Barrel	Day-1 Endcap	Day-100 Barrel	Day-100 Endcap
Pixel	20 μm	50 μm	10 μm	10 μm
SCT	20 μm	50 μm	10 μm	10 μm
TRT	100 μm	100 μm	50 μm	50 μm



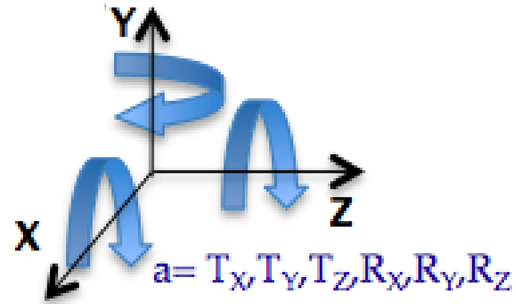
Basic ideas & concepts for alignment

- Basic visualization of the alignment problem
  - Modules are at "unknown" positions. Real hit coordinates are generated by particles that crosses the detector at their "true" location
  - Reconstruction without knowing the real module location. Hits are located at "apparent" positions. Track reconstructions is not accurate
  - After alignment it is possible to have a "residual" misalignment. It will affect the hit positions and the track reconstruction. Hopefully the effect is small



Alignment by χ^2 minimization

- Need to determine 6 alignment parameters per module



$$\boldsymbol{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_{N_A} \end{pmatrix} = \begin{pmatrix} Tx_1 \\ \vdots \\ Rz_1 \\ \vdots \\ Tx_{N_M} \\ \vdots \\ Rz_{N_M} \end{pmatrix}$$

- Define an alignment χ^2 function built from all tracks and hits

$$\boldsymbol{r}_t = \begin{pmatrix} r_{t\ 1} \\ \vdots \\ r_{t\ N_R} \end{pmatrix} \qquad V = \begin{pmatrix} \sigma^2(r_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma^2(r_{N_R}) \end{pmatrix} \quad \rightarrow \quad \chi^2 = \sum_{\forall t} \boldsymbol{r}_t^T V^{-1} \boldsymbol{r}_t$$

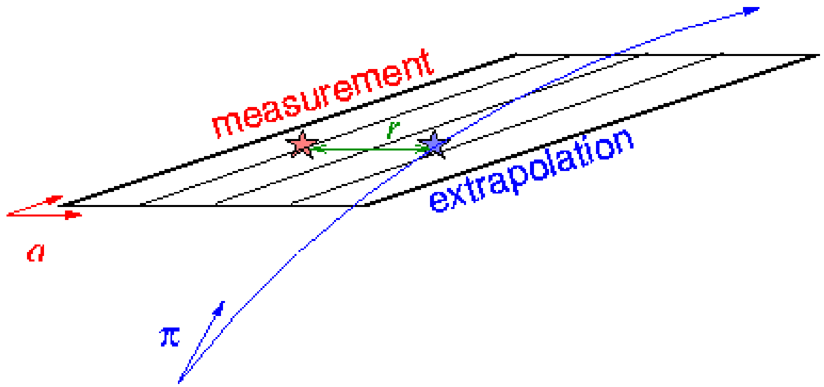
  - Require the minimum condition w.r.t. the alignment parameters

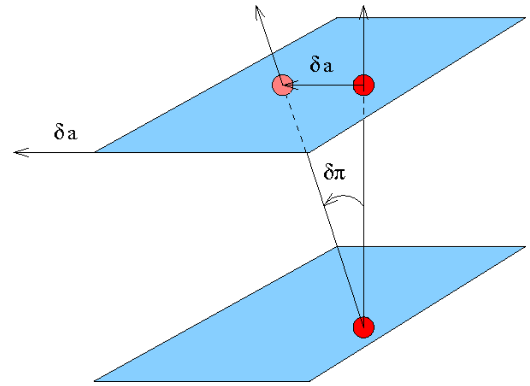
$$\frac{d\chi^2}{d\boldsymbol{a}} = 0 \quad \rightarrow \quad \sum_{\forall t} \left(\frac{d\boldsymbol{r}_t}{d\boldsymbol{a}} \right)^T V^{-1} \boldsymbol{r}_t = 0$$

$$\frac{d\boldsymbol{r}}{d\boldsymbol{a}} = \begin{pmatrix} dr_1/d a_1 & \dots & dr_1/da_{N_A} \\ \vdots & \ddots & \vdots \\ dr_N/d a_1 & \dots & dr_N/da_{N_A} \end{pmatrix}$$

Alignment by χ² minimization

- Now... the residuals derivative contain a nested dependence
  - Residuals depend on track parameters and alignment parameters
  - And track parameters depend on their turn on alignment parameters





  - Mathematically this means:

$$d\boldsymbol{r} = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{a}} d\boldsymbol{a} + \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{\pi}} d\boldsymbol{\pi} \quad \rightarrow \quad \frac{d\boldsymbol{r}}{d\boldsymbol{a}} = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{a}} + \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{\pi}} \frac{d\boldsymbol{\pi}}{d\boldsymbol{a}}$$

    - Actually this is equivalent to a track refit when alignment parameters change

$$\frac{d\boldsymbol{\pi}}{d\boldsymbol{a}} = -\underbrace{\left[\left(\frac{d\boldsymbol{r}}{d\boldsymbol{\pi}}\right)^T V^{-1} \left(\frac{d\boldsymbol{r}}{d\boldsymbol{\pi}}\right)\right]^{-1}}_{\text{track fit matrix}} \left[\left(\frac{d\boldsymbol{r}}{d\boldsymbol{\pi}}\right)^T V^{-1} \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{a}}\right]$$

  - Again, the derivatives can be computed analytically or numerically

Alignment by χ[2] minimization

- Now... use the first order Taylor expansion

$$r = r(a_0) + \left|\frac{\partial r}{\partial a}\right|_{a_0} \delta a$$

  - Neglect second order derivatives
  - Compute track parameters, residuals and derivatives with an initial set of alignment constants a_0

- The alignment solution:

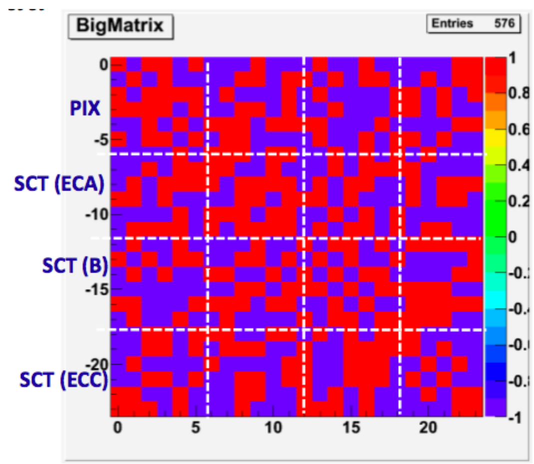
$$\delta a = -\left[\left(\frac{d r}{d a}\right)^T V^{-1}\left(\frac{\partial r}{\partial a}\right)\right]^{-1}\left[\left(\frac{d r}{d a}\right)^T V^{-1} r\right] \quad \rightarrow \quad a = a_0 + \delta a$$

- The alignment matrix can be huge !
  - Size is N_A x N_A
    - ATLAS silicon tracker (pixel + microstrips) 36K x 36K → 4.5 GB
    - CMS tracker: ~100K x 100K (size grows a N_A^2)
  - Inversion time:
    - Tests in ALINEATOR (4-core, 32 GB, parallel) @ IFIC-Valencia
      - Full & dense matrix > 1 day (time grows as ~N_A^3)
      - Correlation matrix of ***a*** available
    - In a commercial PC:
      - Fast inversion of sparse matrix ~1 min
      - No correlation matrix available

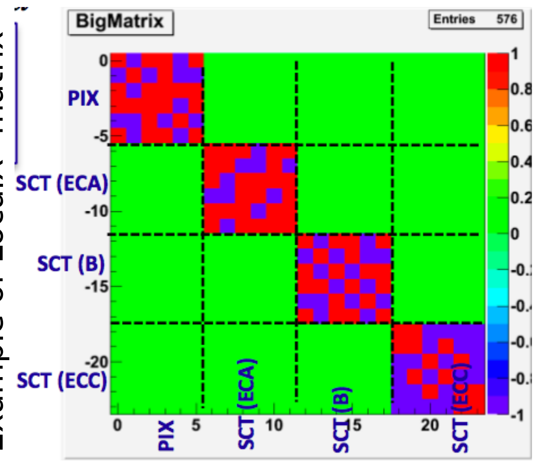
Alignment by χ^2 minimization

- Solving the alignment. Two approaches: Globalχ^2 vs Localχ^2
  - Globalχ^2 :module correlation is taken into account by $d\pi/da$
    - Alignment matrix becomes dense
  - Localχ^2: $d\pi/da = 0$ module correlation is not considered
    - Alignment matrix becomes block diagonal
    - Alignment matrix inversion is not an issue
    - More iterations are needed

Example of GlobalX^2 matrix



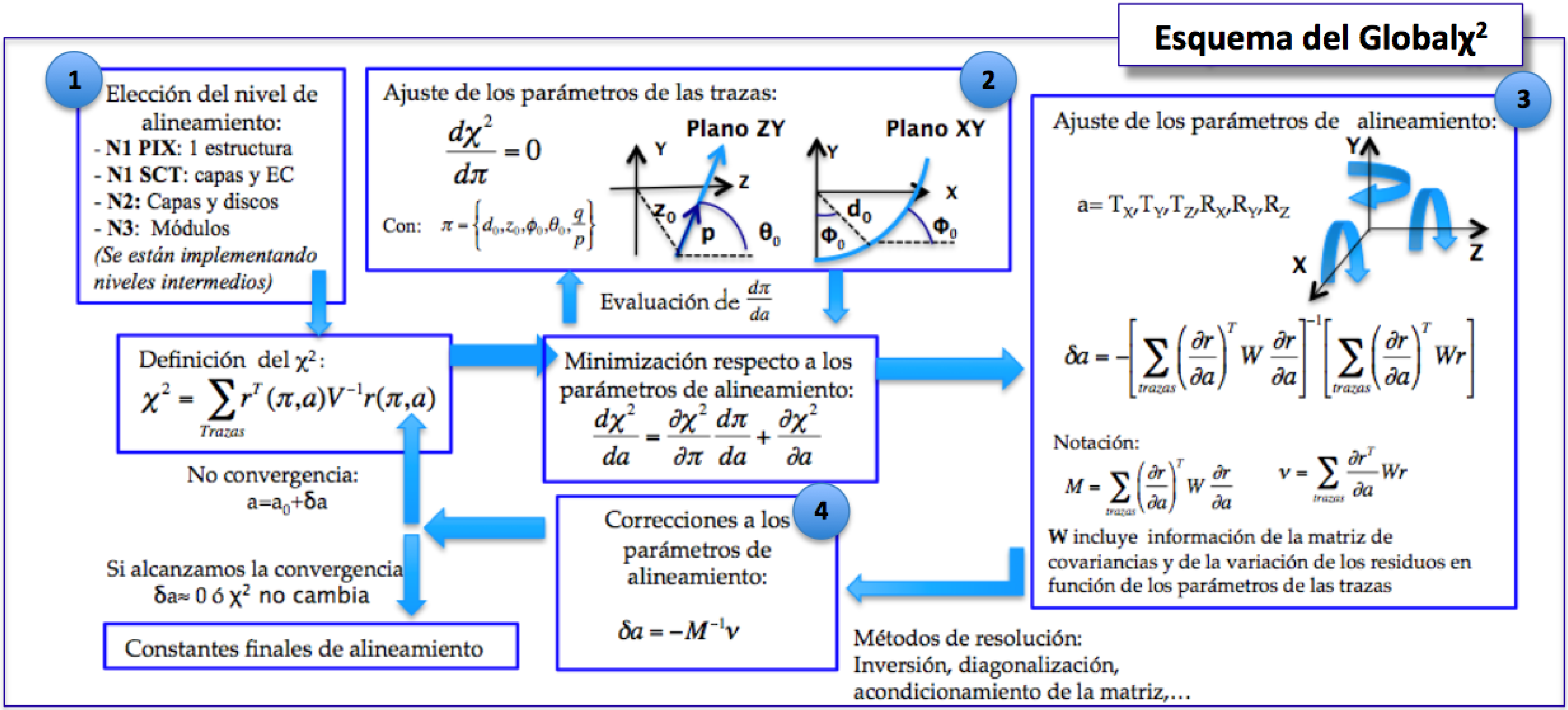
Example of LocalX^2 matrix



- Adding constraints. The alignment χ^2 accepts constraint terms
  - Track parameters: beam spot, invariant masses, E/p for electrons ?
  - Alignment parameters: Assembly survey, online laser survey, soft mode cuts,...

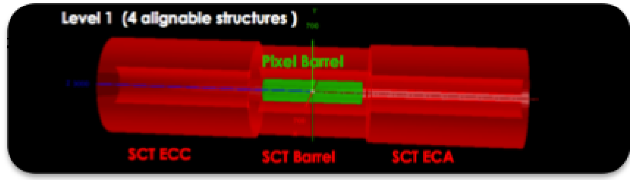
Alignment strategy

- Alignment algorithm is run in an iterative procedure
  - Until convergence is reached
  - Each iteration may take several hours (up to 1 day)

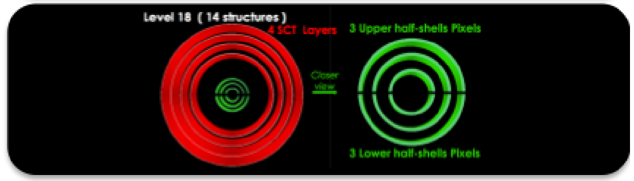


Alignment strategy

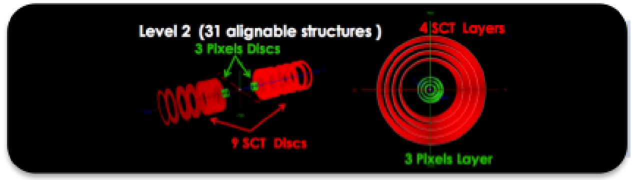
- The alignment procedure mimics the detector assembly structures
  - From large structures
    - PIX, SCT,
    - Barrel, End caps
    - Layers, disks
    - Staves, rings
  - To individual modules
- The size of corrections
  - Large structures
    - mm and mrad
  - Staves
    - 100s microns
  - Modules
    - 10s microns
- Statistics needed:
  - Large structures: O(1000)
  - Staves: O(10,000)
  - Modules: O(1,000,000)



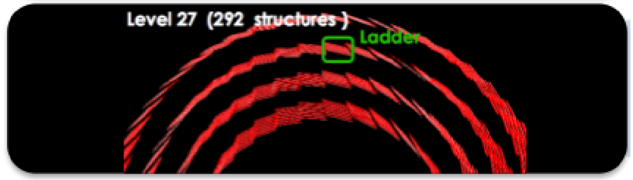
Level 1: 4 struct. → 24 Dofs
PIX: complete detector
SCT: 1 barrel + 2 end caps



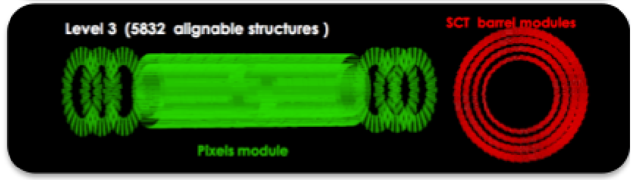
Level 1.8: 14 struct. → 84 Dofs
PIX: (B) 3x2 half layers + 2 EC
SCT: (B) 4 layers + 2 EC



Level 2: 31 struct. → 186 Dofs
PIX: (B) 3 layers + 2x3 EC disks
SCT: (B) 4 layers + 2x9 EC disks



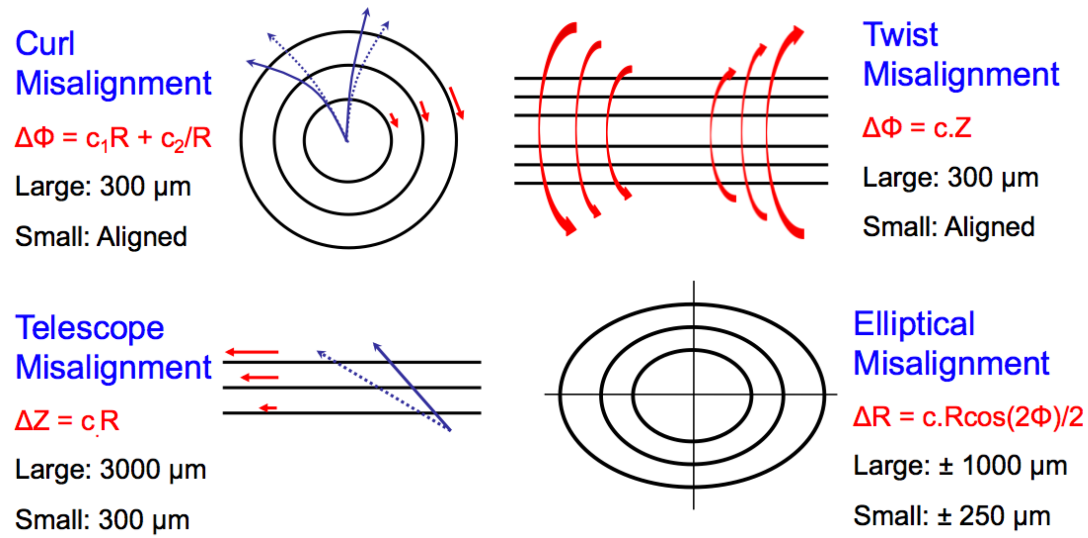
Level 2.7: 292 struct → 1752 Dofs
PIX: (B) 112 staves + 2 EC
SCT: (B) 176 staves + 2 EC



Level 3: 5832 struct → 34992 Dofs
PIX: (B) 1456 + (EC) 2x144
SCT: (B) 2112 + (EC) 2x988

Alignment systematics

- Weak modes: these are solutions of the alignment that do not correspond with real movements, but that preserve the helicoidal path of the tracks, leaving the track χ^2 almost unchanged
- Examples of weak modes:



- Material effects:
  - In order to achieve a resolution of the alignment corrections down to 1 micron one needs to consider closely the material effects in the track reconstruction.
  - The material description must be accurate and all operational conditions under control
  - Detector deformation: out of plane twisting and bending (planar silicon devices), wire sag (gas systems)

Alignment summary

- The goal of the ID alignment is to determine the position of the tracking modules with enough precision for the physics analysis
  - This requires precision below 10 microns (ultimate goal 1 micron)
  - Determination of almost 40K ATLAS & 100K CMS degrees of freedom
    - 6 per module (Tx, Ty, Tz, Rx, Ry, and Rz)
- Track based alignment algorithms can reach good precision
  - Combination almost mandatory with survey constraints
  - Track parameters constraints
- Study of random and systematic deformations is difficult to tackle

- ATLAS & CMS alignment of tracking systems ready for first LHC collisions

Thanks to: Carlos Escobar, Vicente Lacuesta and Regina Moles

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