High Density QCD-Matter

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Universidad de Oviedo

Outline

⇒ Part I

- ✓ Motivation. QCD & the QCD vacuum
- ✓ QCD at high temperature or density: Quark Gluon Plasma

⇒ Part II

- ✓ Heavy Ion collision experiments
- ✓ Relevant findings at RHIC

Strong interactions are responsible for 99% of (visible) matter in the Universe

Electromagnetism

Microscopic theory: QED (p, e, γ)

⇓

Macroscopic, collective behavior:

- Phase transitions: gas, solid, fluid, superfluid ...
- Condensed / solid state physics: Insulators, semi-conductors, ferromagnets, glasses ...
- Chemistry ... industry

Strong interactions

Microscopic theory: QCD (quarks, gluons)

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Strong interactions

Microscopic theory: QCD (quarks, gluons)

$\Downarrow$

Macroscopic, collective behavior:

- What are the phases of QCD ?

⋮

- Is a color-chemistry possible?
- Are there color-superconductors?
- Color-industry?

$\Downarrow$

Study of QCD matter at high density or temperature

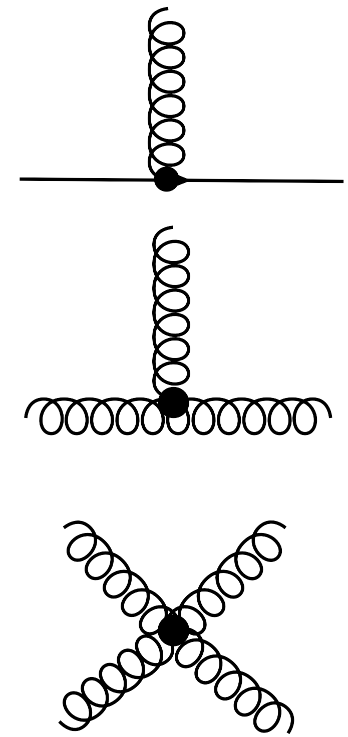
Strong interactions are responsible for 99% of (visible) matter in the Universe

Microscopic theory ⇒ Quantum Chromodynamics

$$\mathcal{L}_{QCD} = \sum_{flavors} \bar{q}_f \, (i \not{D} - m_f) \, q_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \ldots$$

quarks $q_f^{\alpha,\,a} \rightarrow \begin{cases} \alpha = 1, \ldots 4 & \text{Lorentz index} \\ a = 1 \ldots N_c = 3 & \text{Color index} \\ f = \text{u, d, s, c, b, t} & \text{Flavor index} \end{cases}$

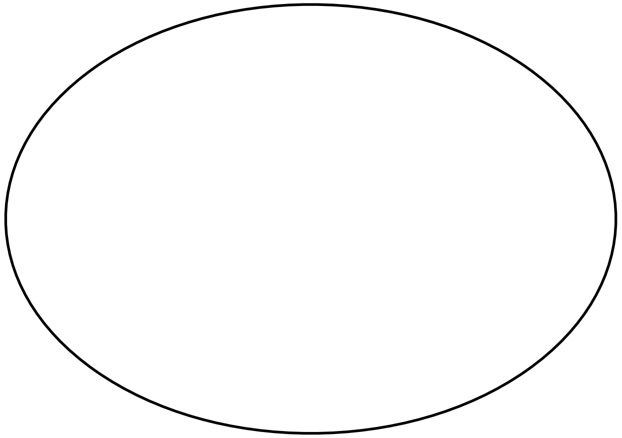
gluons $A^{\mu,a} \rightarrow \begin{cases} \mu = 1, \ldots 4 & \text{Lorentz index} \\ a = 1 \ldots N_c^2 - 1 = 8 & \text{Color index} \end{cases}$



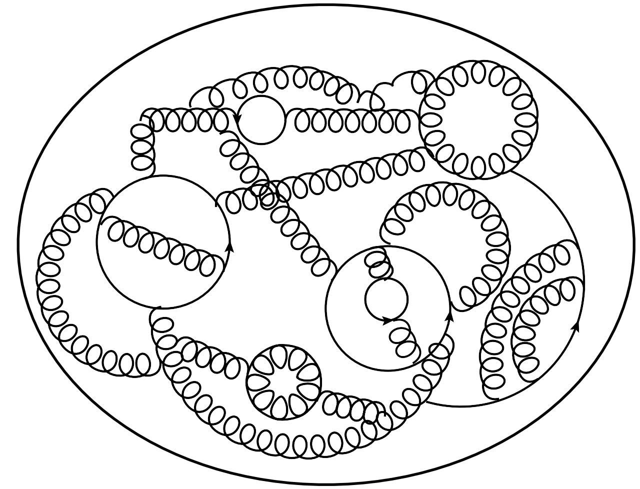
Gauge symmetry: SU(N_c=3) (non-abelian)

+2/3	u (3 MeV)	c (1.2 GeV)	t (171 GeV)
-1/3	d (5 MeV)	s (105 MeV)	b (4.2 GeV)

the NOTHING



the QCD VACUUM



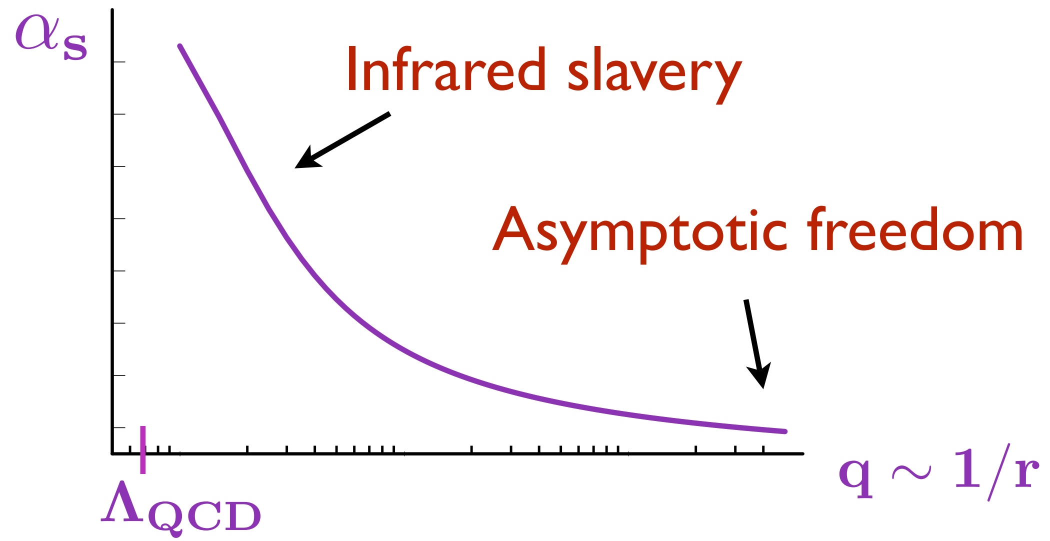
The QCD ground state has a complicated structure:

- It anti-screens color charges (running coupling and asymptotic freedom)
- It has negative energy density
- It is confining: quarks and gluons do not exist as free states
- It breaks a few symmetries of the QCD Lagrangian: chiral, conformal
- It has a non-trivial topological structure: Instantons ...
- It has quark and gluon condensates...

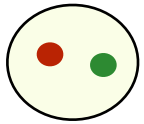
⇒ The QCD coupling runs:

$$\alpha_s(q^2) = \frac{4\pi}{\beta \ln\left(q^2/\Lambda^2_{QCD}\right)}$$

$$\beta = \frac{11\,N_c - 2\,N_f}{3}$$

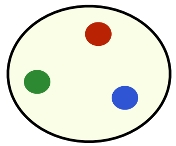


⇒ Confinement: Color is confined within hadrons; $R_{hadron} \sim \Lambda^{-1}_{QCD} \sim 1\,\mathrm{fm}$



mesons (q$\bar{\text{q}}$)

$\pi,\ K,\ \rho\ldots$



baryons (qqq)

$p,\ n,\ \Lambda' s\ldots$

Most of the hadron's masses is due to interaction:

$$m_{proton}(uud) \sim 1\,\mathrm{GeV}\,; \quad 2\,m_u + m_d \sim 10\,\mathrm{MeV}$$

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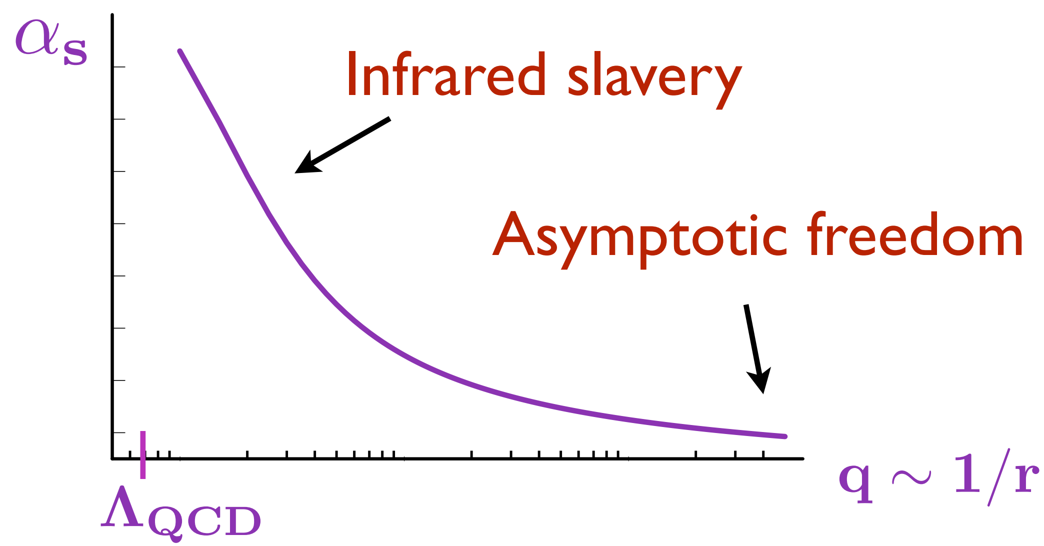

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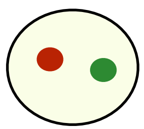

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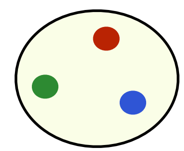

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Would a high-temperature (density) QCD sytem allow free quarks and gluons?

$$\text{if } T \gg \Lambda_{QCD} \quad \text{then} \quad \alpha_s(T) \ll 1$$

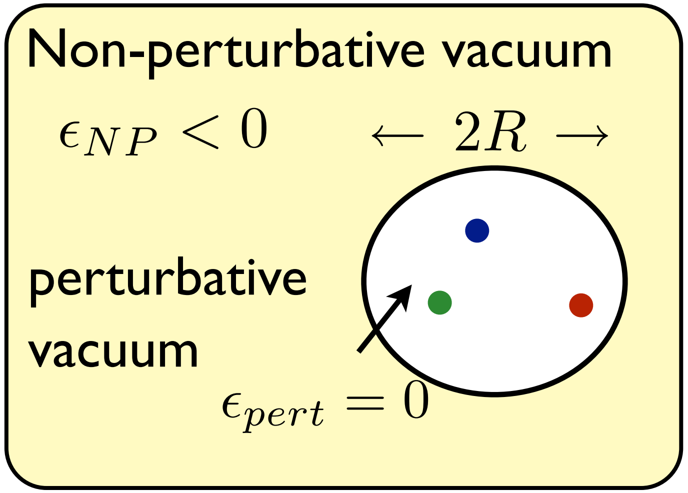
YES!!

⇒Bag model: Hadrons are "droplets" of perturbative vacuum with quasi free quarks and gluons inside:

$$H_{bag} = H_{kin} + H_{bag} + \dots \approx \frac{x}{R} + \frac{4}{3}\pi R^3 B + \dots$$

Bag constant

$$B \sim \epsilon_{pert} - \epsilon_{Non-pert} \sim (250\,\mathrm{MeV})^4$$



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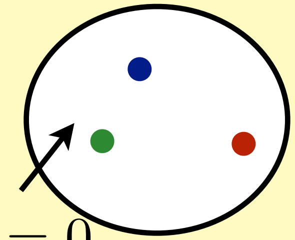
Non-perturbative vacuum

$\epsilon_{NP} < 0$ $\quad \longleftarrow 2R \longrightarrow$

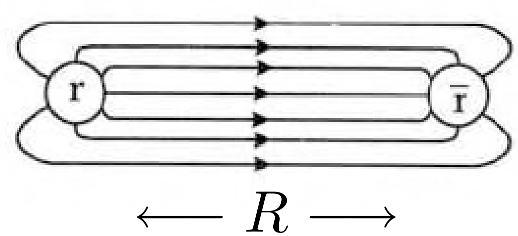


perturbative vacuum

$\epsilon_{pert} = 0$

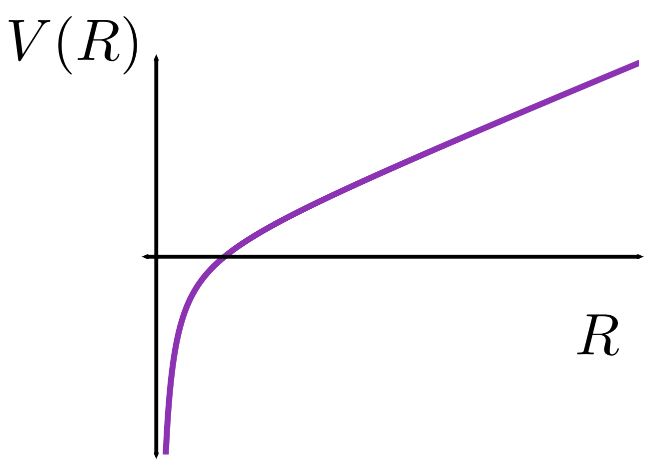
⇒Potential models. Lines of color field are confined to flux tubes or strings

$$V(R) = -\frac{\alpha_{eff}}{R} + K\,R$$



String tension:

$$K \sim (420\,\mathrm{MeV})^2 = 900\,\mathrm{MeV\,fm}^{-1}$$



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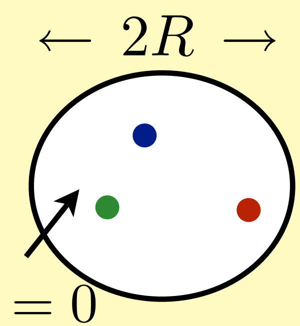

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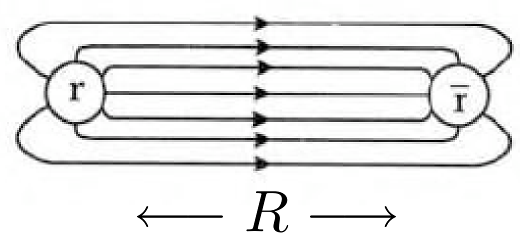

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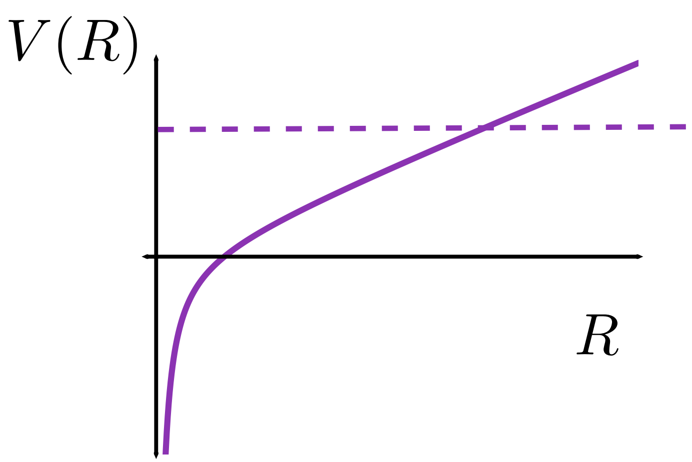
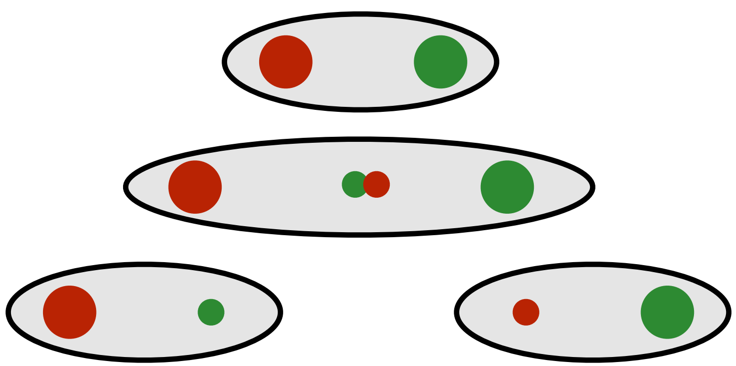
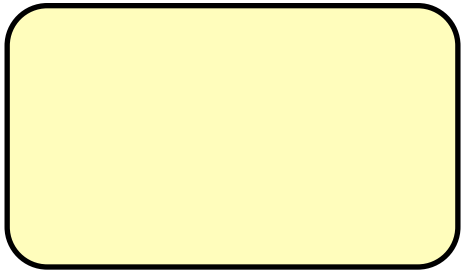


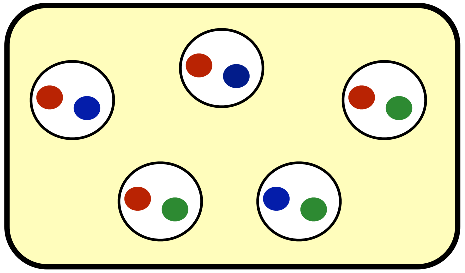
With dynamical quarks, the string breaks:



Vacuum at T=0



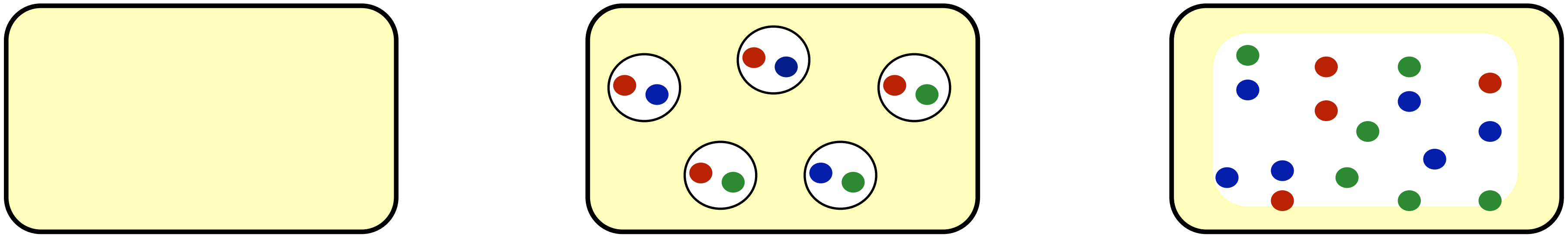
Pion Gas T>0



⇒ Pressure and energy density of ideal Bose (and Fermi) massless gas

Pion gas: $$p_\pi \approx d_\pi \frac{\pi^2}{90} T^4 , \quad \epsilon_\pi = 3\, p_\pi , \quad d_\pi = 3 \ (\pi^\pm, \pi^0)$$

Vacuum at T=0 | Pion Gas | Quark-Gluon Plasma



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QGP:

$$p_{QGP} \approx d_{gq\bar{q}} \frac{\pi^2}{90} T^4 - B , \quad \epsilon_{QGP} \approx d_{gq\bar{q}} \frac{\pi^2}{30} T^4 + B$$

$$d_{gq\bar{q}} = d_g + \frac{7}{8} d_{q\bar{q}} = 2_s \cdot (N_c^2 - 1) + \frac{7}{8} \cdot 2_{q\bar{q}} \cdot 2_s \cdot N_c \cdot N_f = 37 \; (N_f = 2)$$

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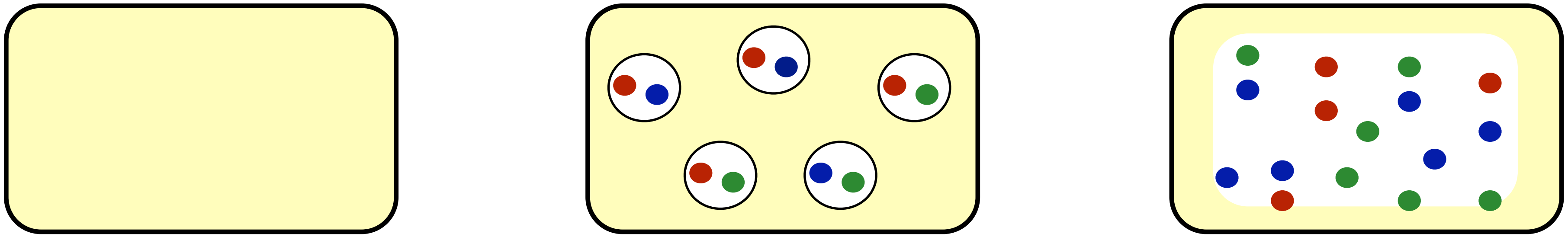

$T < T_c$ | $T > T_c$

⇒ Pressure and energy density of ideal Bose (and Fermi) massless gas

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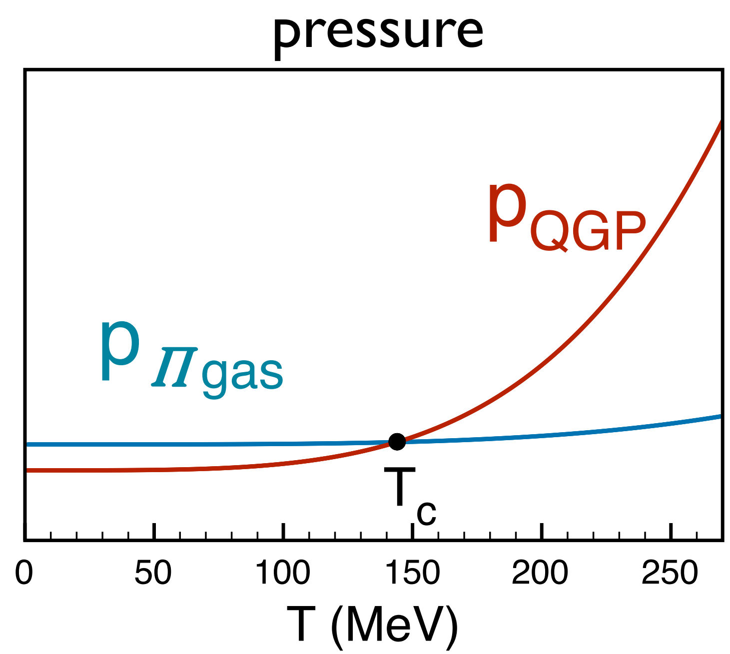
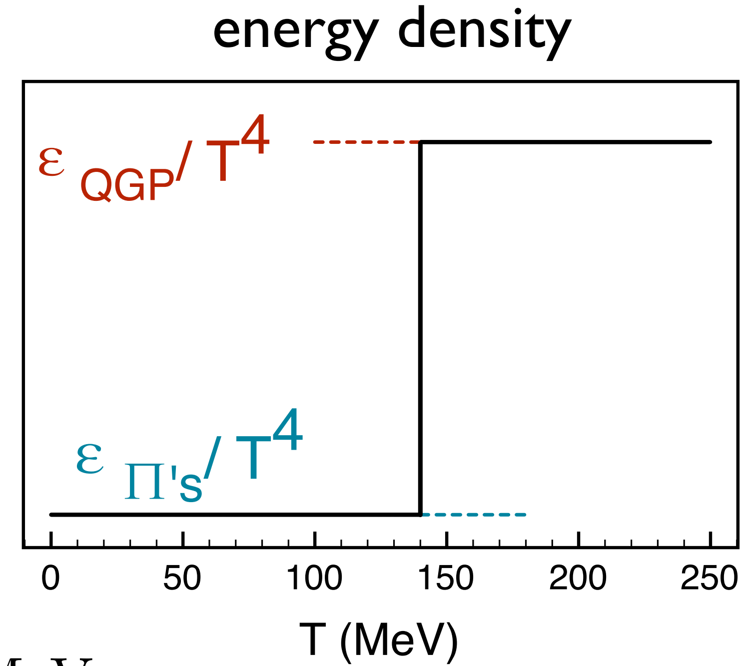
⇒ At T=Tc the pressure of the QGP becomes larger than that of the pion gas

$$p_{QGP}(T_c) = p_\pi(T_c)$$

$$T_c = \left(\frac{90}{\pi^2 (d_{gq\bar{q}} - d_\pi)} B \right)^{1/4} \approx 0.7\, B^{1/4} \approx 140\,\text{MeV}, \quad \text{for } B^{1/4} = 200\,\text{MeV}$$





$$T_c \approx 140\,\mathrm{MeV}$$

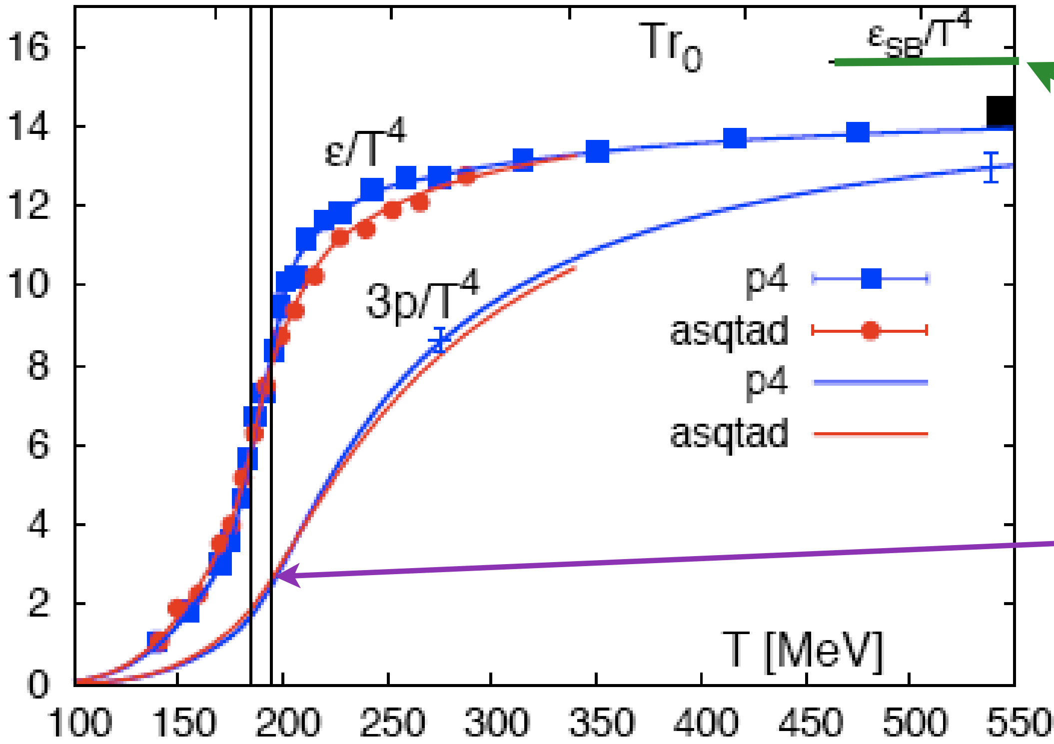
Latent heat of the phase transition: $L = \epsilon_{QGP}(T_c) - \epsilon_{\pi' s} \sim 4B \sim 1\mathrm{GeV\,fm}^{-3}$

Energy density of nuclear matter $\epsilon_{nm} \sim 0.15\,\mathrm{GeV\,fm}^{-3}$

QGP	$T_c^{QGP} \approx 170\,\mathrm{MeV} \sim 2 \cdot 10^{12}\,\mathrm{Kelvins}$
Sun core	$T_{Sun} \sim 1.5 \cdot 10^7\,\mathrm{Kelvins}$
Córdoba	$T_{\text{Córdoba}} \sim 10^3\,\mathrm{Kelvins}$

Results from Lattice QCD

Energy density & pressure



Stephan-Boltzman (ideal gas) limit

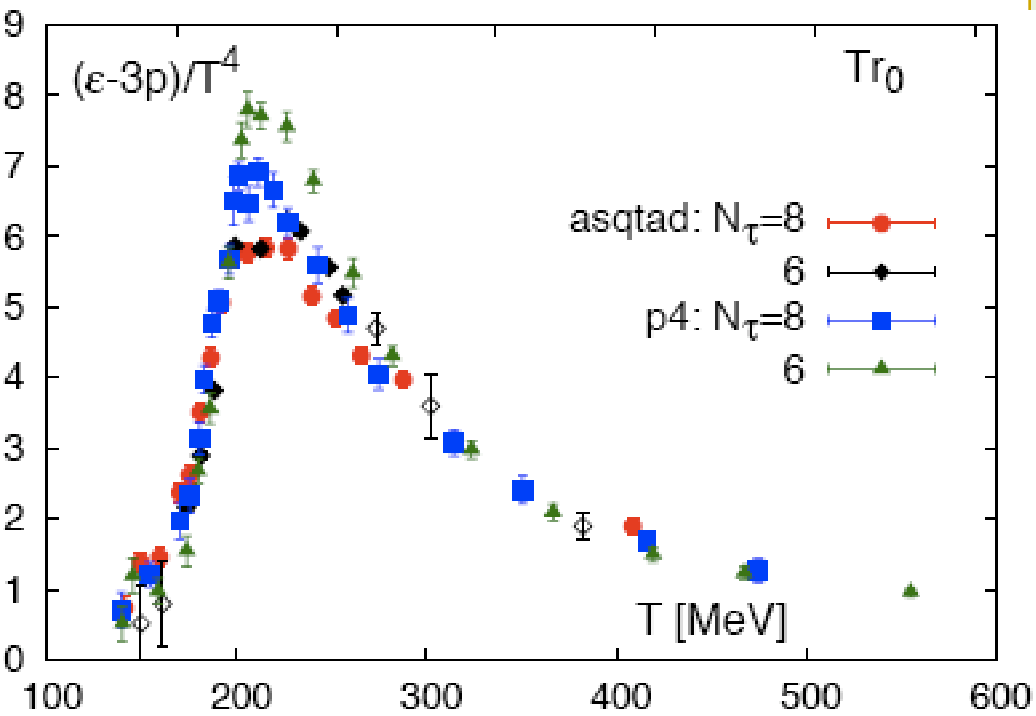
"Explosion" of degrees of freedom

$$T_c \approx 170 \div 180\,\mathrm{MeV}$$

For an ideal gas $\epsilon = 3p \sim T^4$

The "trace anomaly" $T^\mu_\mu = \epsilon - 3p$

is a measure of the interaction
(and also of the degree of violation of scale symmetry)



An alternative view: Broken symmetries and phase transitions

⇒ QCD with massless quarks can be decomposed into right- and left-handed sectors

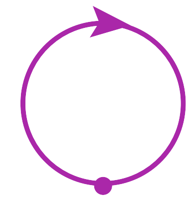
$$\mathcal{L}_{quarks} = \bar{q}_L\, i\, \not{D}\, q_L + \bar{q}_R\, i\, \not{D}\, q_R \qquad\qquad q_{L(R)} = \frac{1 \mp \gamma^5}{2}\, q$$

It is invariant under $SU_L(N_f) \times SU_L(N_f)$; $\begin{pmatrix} u \\ d \end{pmatrix}_{L(R)} \mapsto \exp\left[i\, \theta^a_{L(R)}\, \lambda^a\right] \begin{pmatrix} u \\ d \end{pmatrix}_{L(R)}$

⇒ Chiral symmetry is spontaneously broken in the vacuum (dynamical origin of mass in QCD):

Quark (chiral) condensate:

$$\langle 0|\bar{q}\, q|0\rangle = \langle 0|\bar{q}_L\, q_R + \bar{q}_R\, q_L|0\rangle \approx -(240\,\mathrm{MeV})^3$$



The chiral condensate can be regarded as an order parameter for the phase transition

$$M_q \sim \frac{m_{hadron}}{N_{quarks}} \propto \qquad \langle 0|\bar{q}q|0\rangle = \begin{cases} \neq 0, & \text{for } T < T_c \\ = 0, & \text{for } T > T_c \end{cases}$$

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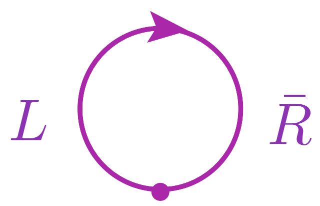

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⇒ Other symmetries: Center symmetry Z(Nc) for Polyakov loops (infinitely heavy masses)

$$L(\vec{x}) = \frac{1}{N_c} \, \mathrm{tr} \exp \left[i \, g \int_0^{\frac{1}{T}} A_4(\tau, \vec{x}) \, d\tau \right] \qquad \langle 0 | L(\vec{x}) | 0 \rangle = \begin{cases} = 0, & \text{for } T < T_c \\ \neq 0, & \text{for } T > T_c \end{cases}$$

An alternative view: Broken symmetries and phase transitions

⇒Other symmetries: Center symmetry Z(Nc) for Polyakov loops. It is the order parameter in the case of infinitely heavy masses or pure gluodynamics

$$L(\vec{x}) = \frac{1}{N_c} \mathrm{tr} \exp \left[i\, g \int_0^{\frac{1}{T}} A_4(\tau, \vec{x})\, d\tau \right]$$

$$z \in Z(N_c) \Rightarrow z = \exp \left[i \frac{2\pi n}{N_c} \right]$$

$$\langle L(\vec{x}) \rangle \sim \exp\left[-F_Q/T\right]$$

Physically it is related to the (free) energy of a single quark

The QCD action is invariant under Z(Nc) transformations; the Polyakov loop is not:

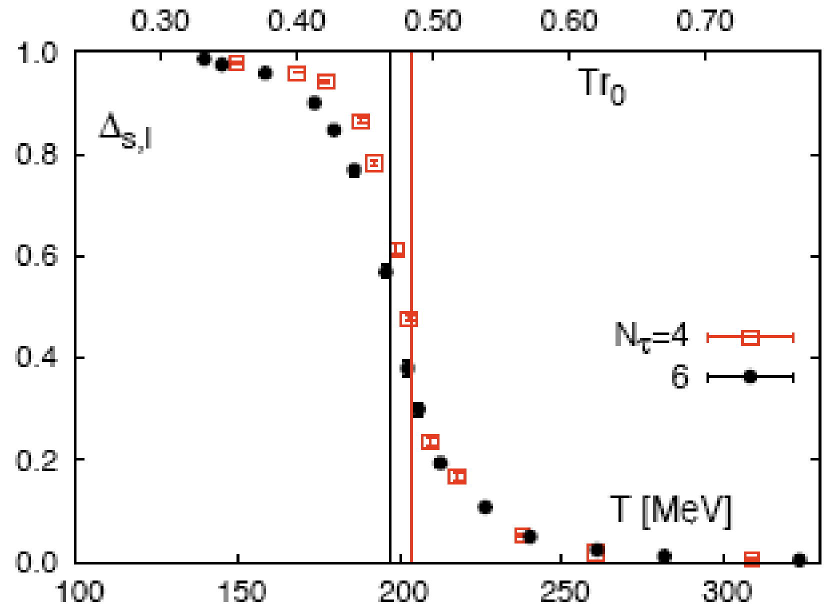
$$\langle L(\vec{x}) \rangle \rightarrow z\, \langle L(\vec{x}) \rangle$$

$$\langle 0 | L(\vec{x}) | 0 \rangle = \begin{cases} = 0, & \text{for } T < T_c \\ \neq 0, & \text{for } T > T_c \end{cases}$$

Results from lattice QCD

- The chiral symmetry (Z(Nc)) is restored (broken) above the phase transition:
- The inclusion of finite (bare) quark masses makes the phase transition smooth (crossover)

$\langle q\bar{q} \rangle$



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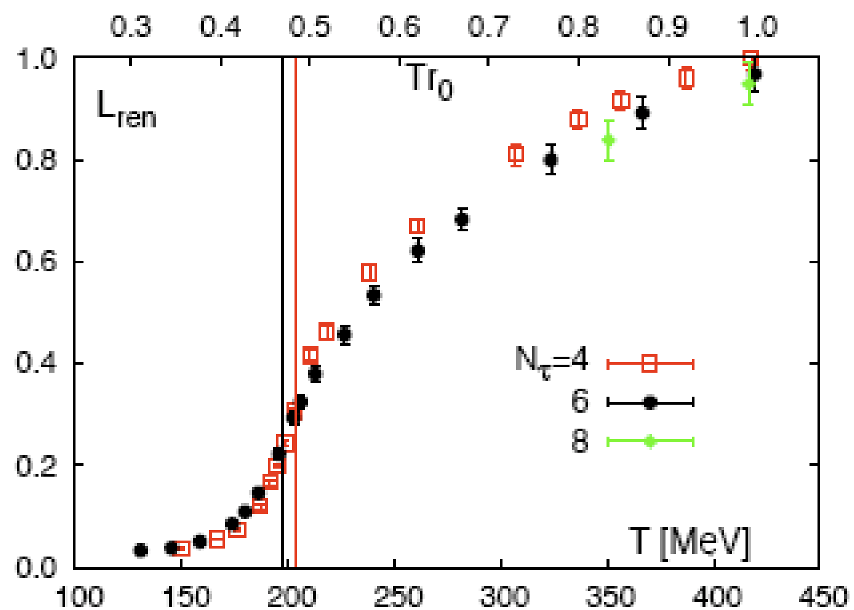
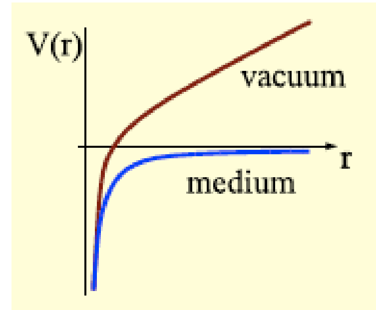
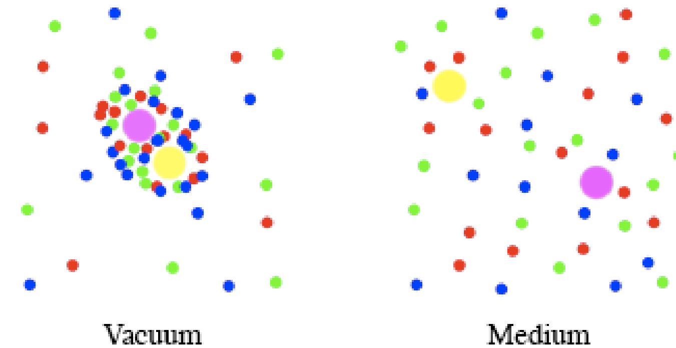


Debye screening of the heavy quark potential in the QGP phase

- The presence of free quarks and gluons around a heavy quark pair screens the interaction.

- The string tension tension goes to zero





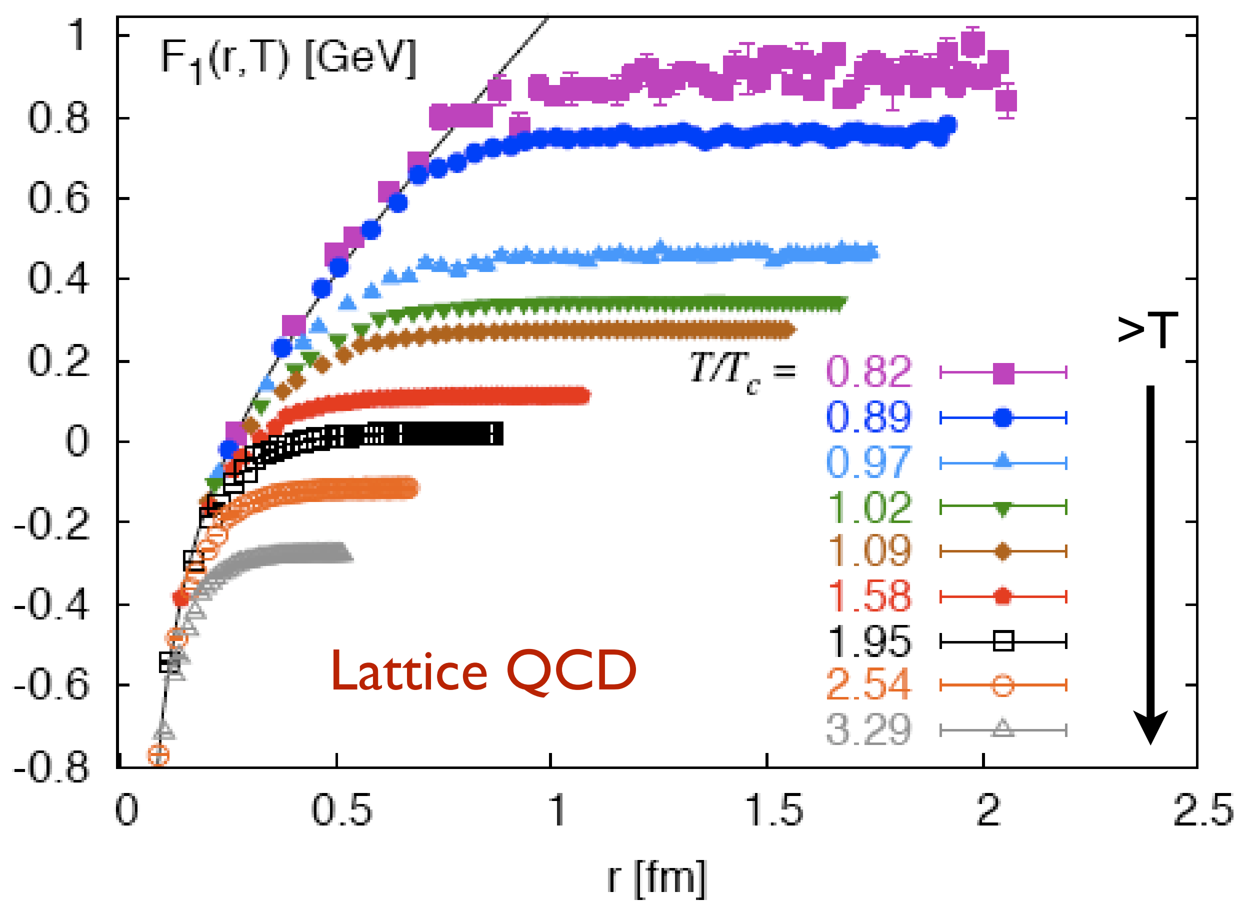
$$V(r,T) \approx -\frac{\alpha_{eff}}{r} \exp[-m_D\, r] + K(T)\, r$$

Debye mass

$$m_D^2 = \frac{N_c + \frac{1}{2} N_f}{3} g^2 T^2$$

effective string tension

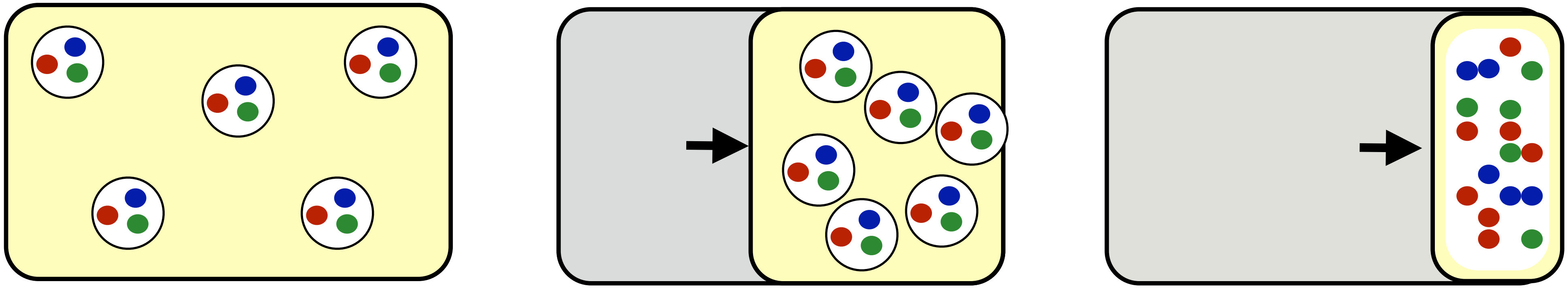
$$K(T) \to 0 \;\; \text{for } T >> T_c$$



⇒Other way for the QGP: compressing nuclear matter at low temperatures

Baryon number density ~ Baryochemical potential

$$n_B = \frac{1}{3}\frac{N_q - N_{\bar{q}}}{V} = d \cdot \frac{T^3}{6}\left[\frac{\mu_B}{T} + \frac{1}{\pi^2}\left(\frac{\mu_B}{T}\right)^3\right]$$



$\mu_B < \mu_{Bc}$ $\mu_B > \mu_{Bc}$

Pressure of a Fermi gas:

$$p_F = d \cdot \frac{T^4}{3}\left[\frac{7\pi^2}{120} + \frac{1}{4}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{8\pi^2}\left(\frac{\mu_B}{T}\right)^4\right]$$

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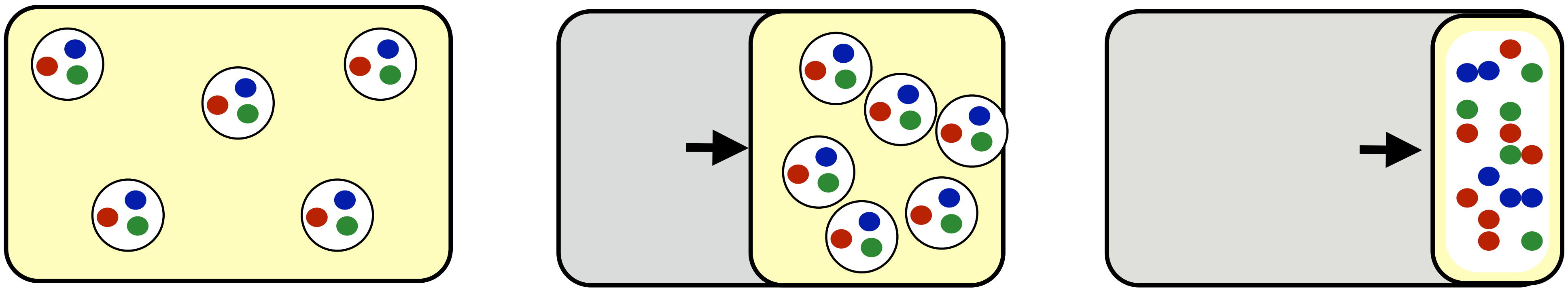
$$n_B = \frac{1}{3}\frac{N_q - N_{\bar{q}}}{V} = d \cdot \frac{T^3}{6}\left[\frac{\mu_B}{T} + \frac{1}{\pi^2}\left(\frac{\mu_B}{T}\right)^3\right]$$



$\mu_B < \mu_{Bc}$ $\qquad$ $\mu_B > \mu_{Bc}$

Pressure of a Fermi gas:

$$p_F = d \cdot \frac{T^4}{3}\left[\frac{7\pi^2}{120} + \frac{1}{4}\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{8\pi^2}\left(\frac{\mu_B}{T}\right)^4\right]$$

Critical baryochemical potential for the QGP phase transition (T=0)

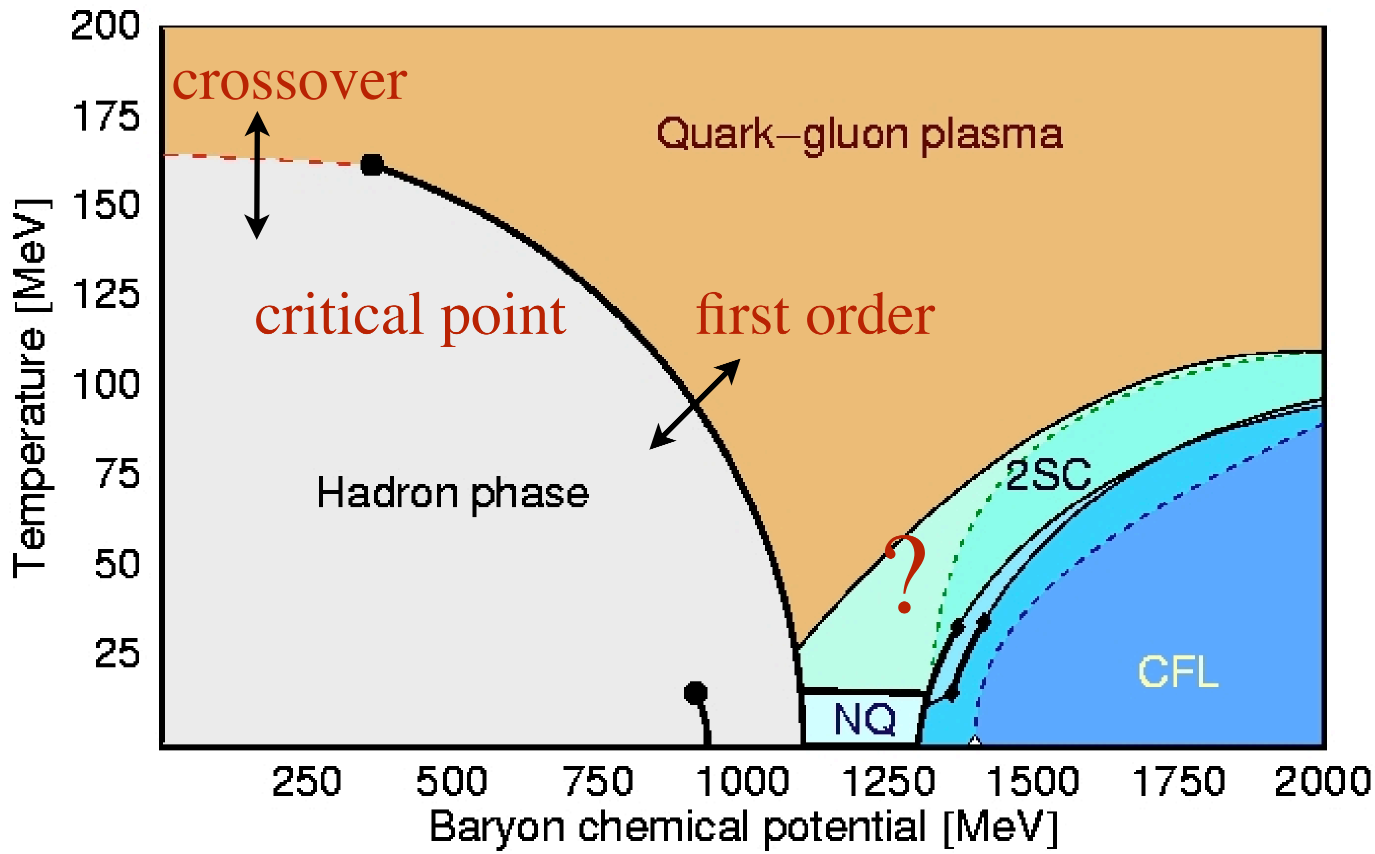
$$p_{q\bar{q}}(\mu_{Bc}) = B \Longrightarrow \mu_{Bc} \approx 3\sqrt{\pi}\, B^{1/4} \approx 1.1 \text{ GeV}$$

Nuclear matter: $\mu_{B\,nm} \approx 0.9 \text{ GeV}$

Putting all together: The phase diagram of QCD

- At low the phase transition is smooth crossover between hadron gas and QGP. *More like melting butter*

- At larger the transition becomes first order. Existence of a critical point. *More like water-vapor transition*



- A number of phases, Color Superconductivity (2SC), Color Flavor Locked (CFL) ... have been proposed. Lattice methods not reliable ready in this regime ....

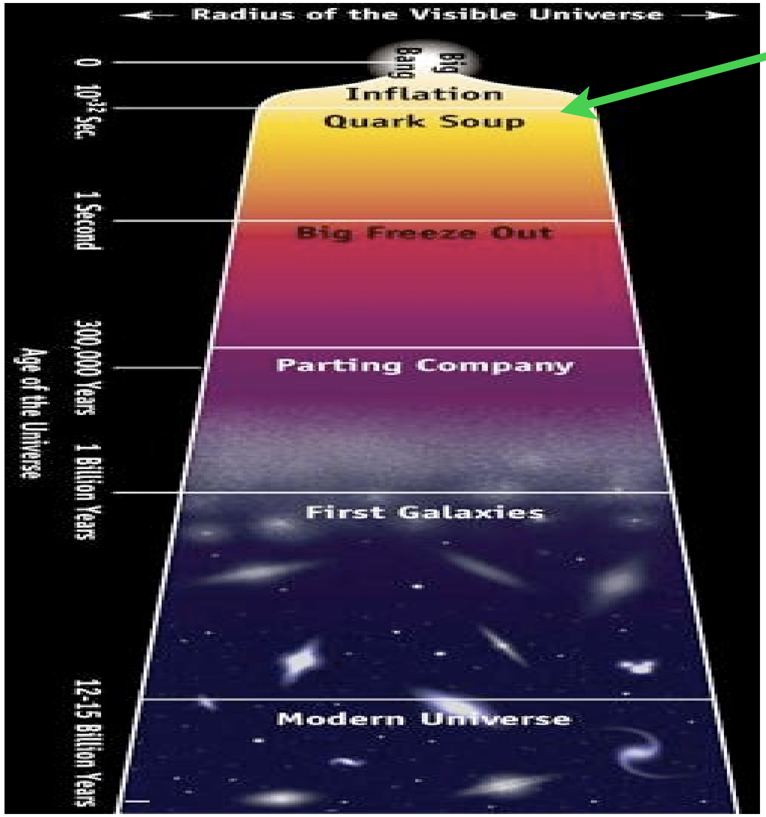
Where to find the QGP?

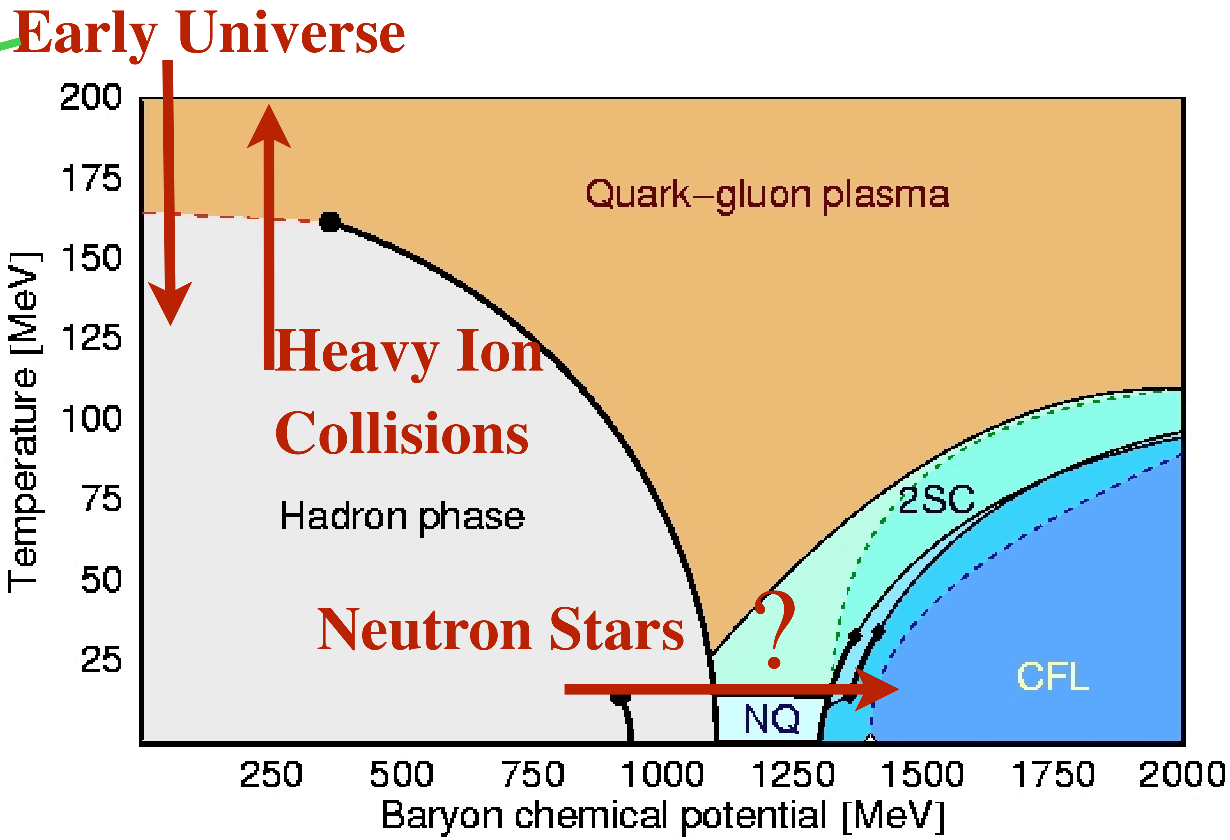
⇒ Heavy ion collisions

⇒ Core of neutron stars may be composed "exotic" quark matter

$$M_{NS} \sim 1 \div 2\, M_{Sun}\,; \quad R_{NS} \sim 10\,\mathrm{km}$$

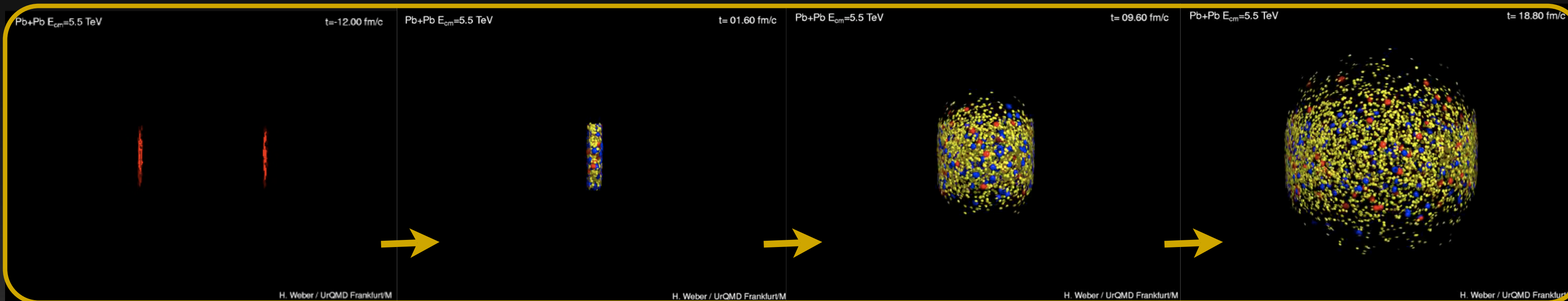
⇒ Early Universe: The temperature of the Universe at time $10^{-4} \sim 10^{-5}$ seconds was $T_{univ} \sim 200$ MeV. It went through a phase transition from quarks and gluons to hadrons

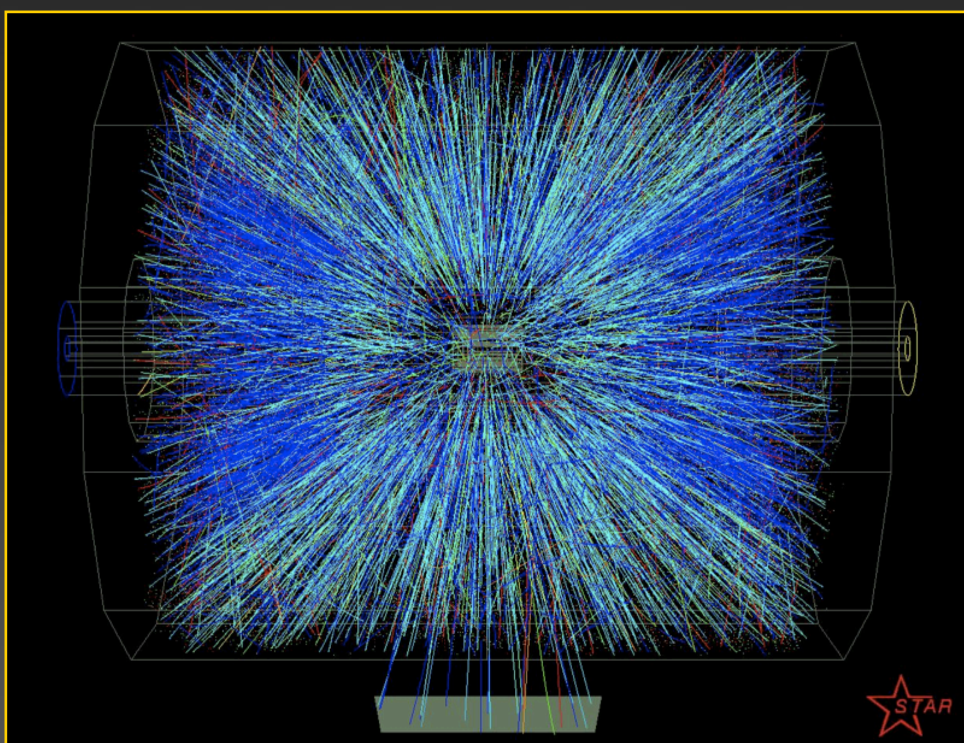




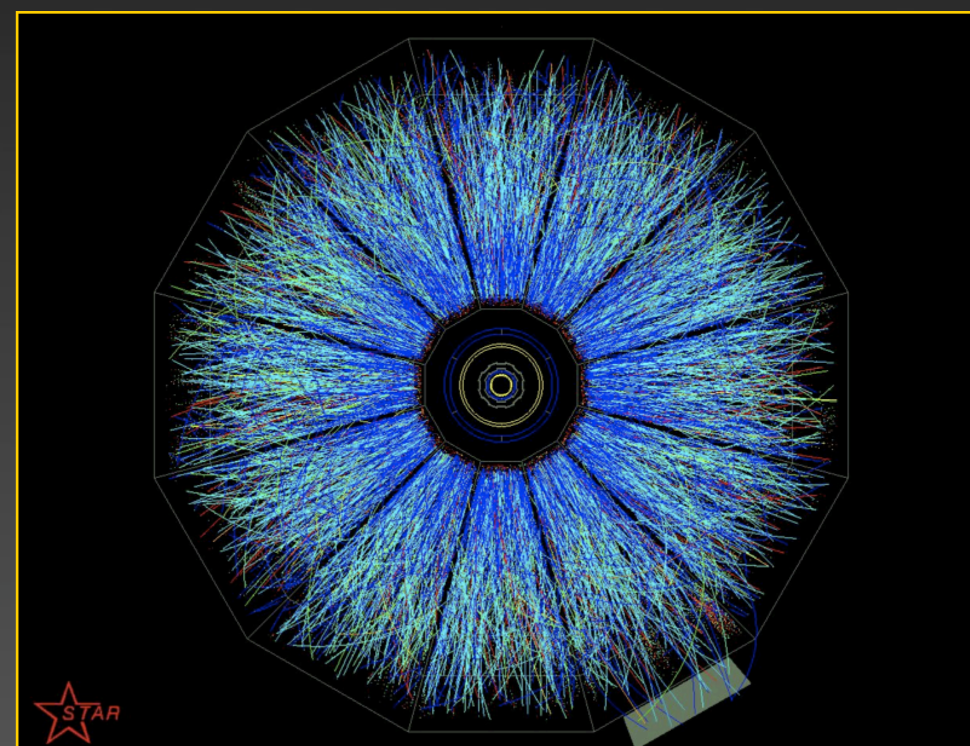
Ultra-relativistic heavy ion collisions

Searching for the Quark Gluon Plasma





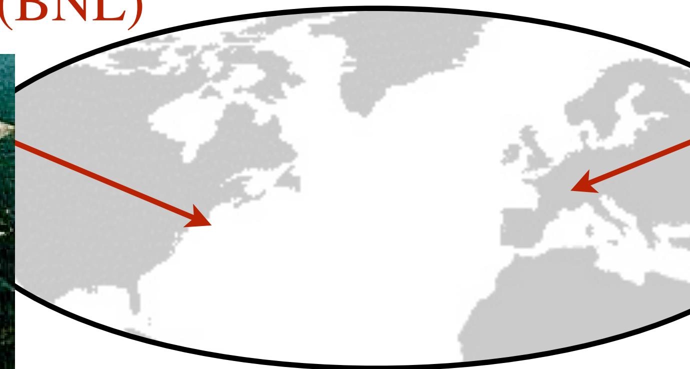
side view



front view

Relativistic Heavy Ion Collider (RHIC)
Alternating Gradient Synchrotron (AGS)
@ Brookhaven National Lab (BNL)

Large Hadron Collider (LHC)
Super Proton Synchrotron (SPS)
@ CERN



	Lab	years	$\sqrt{s_{NN}}$ (GeV)	
AGS	BNL	87/99	5	
SPS	CERN	86/02	17	
RHIC	BNL	01/??	200	Au-Au, d-Au, p-p, Cu-Cu
LHC	CERN	??/??	5500	Pb-Pb, p-Pb, p-p

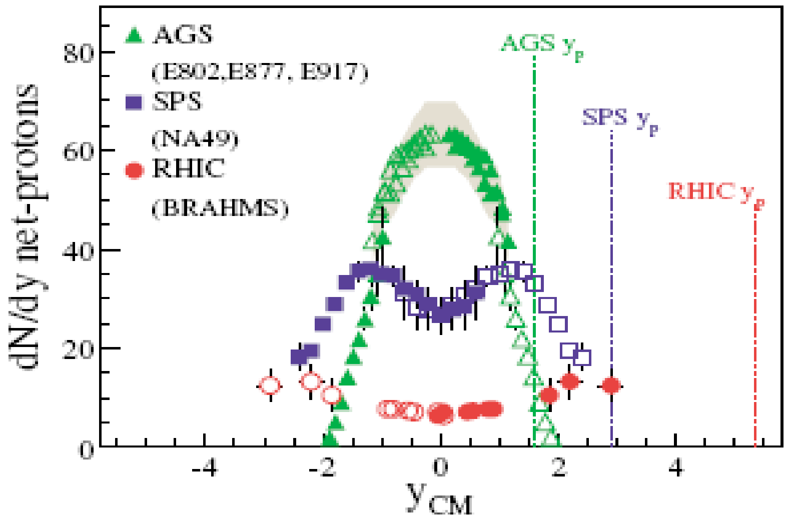
- First hints of QGP formation at SPS. More conclusice evidence obtained at RHIC
- Of the 4 big experimental collaborations at the LHC, one (ALICE) is fully dedicated to HIC. Other two (ATLAS and CMS) will perform related measurements

Locating HIC experiments on the QCD phase diagram:

- The baryon density in the midrapity region decreases with increasing collison energy

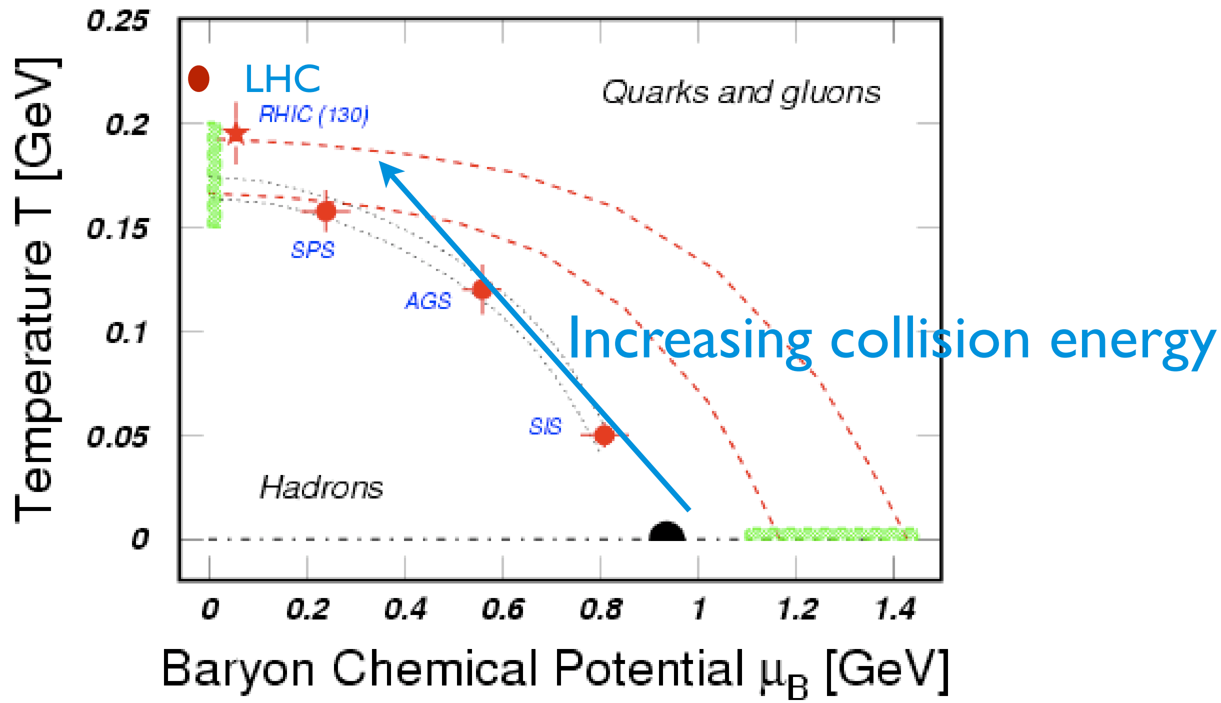
$$\eta = \ln \frac{p_0 + p_z}{p_0 - p_z}$$

High energy:
valence quarks are not
slowed down by the collison

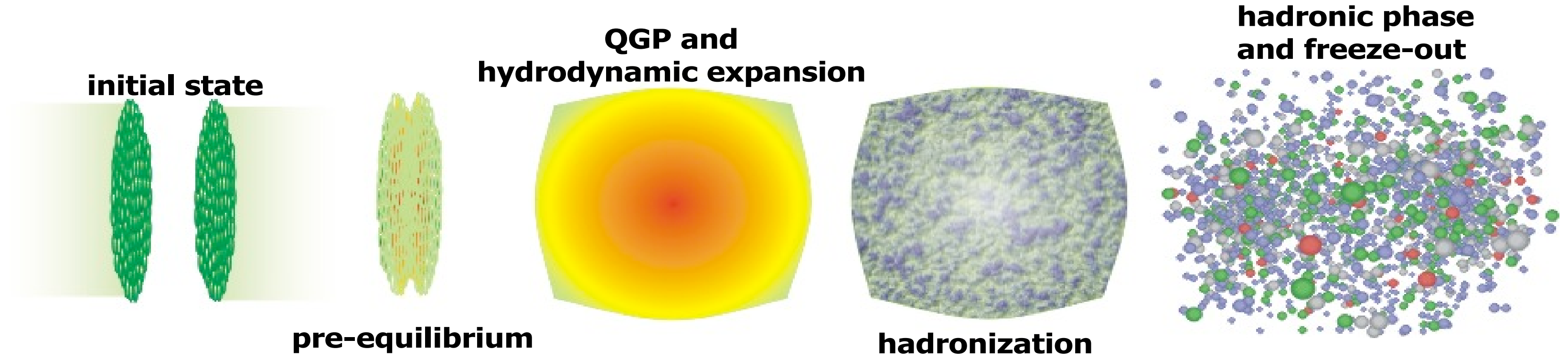


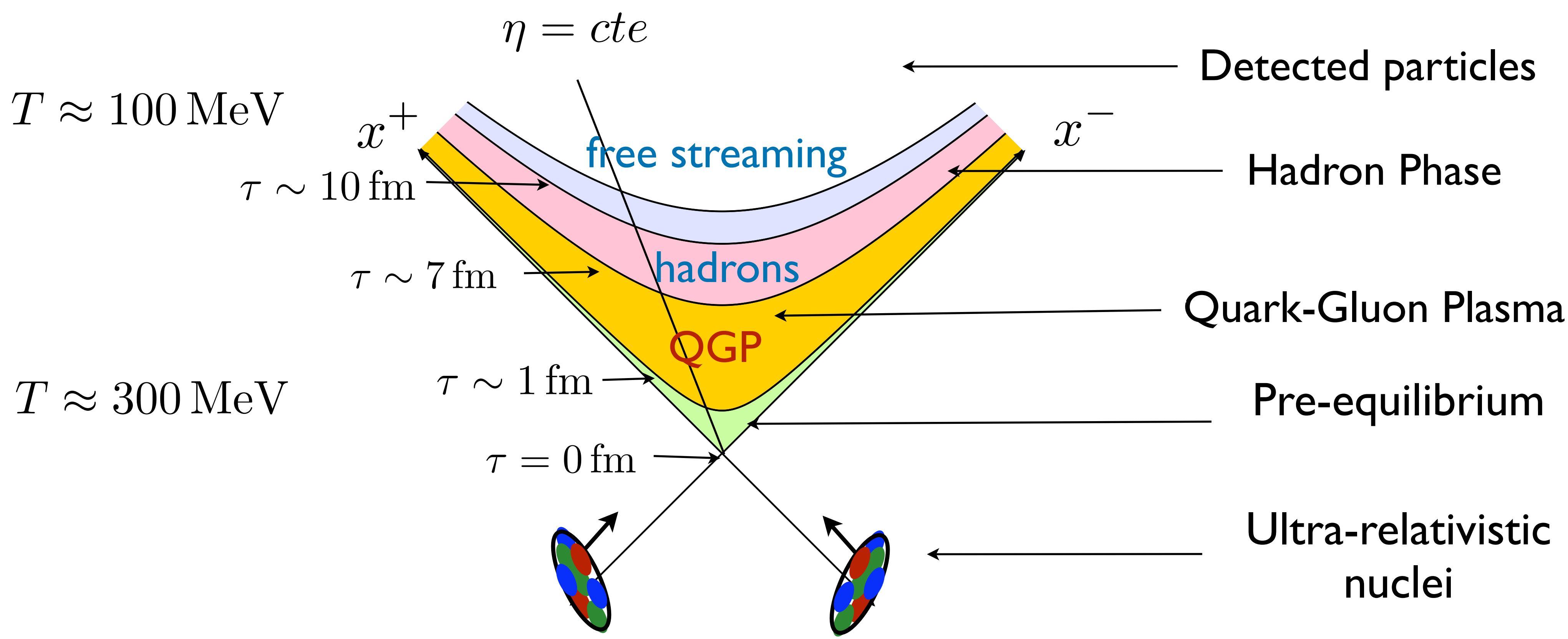
increasing collision energy

- The temperature increases with collision energy



Space-time view of heavy-ion collisions

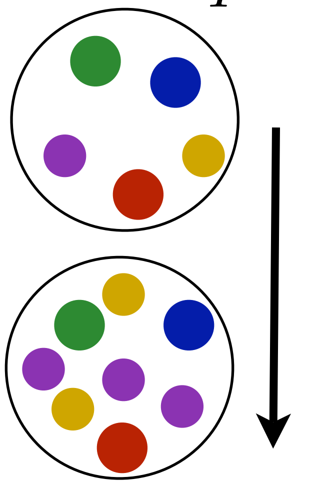




We lack of a unified description of the collision dynamics at all times

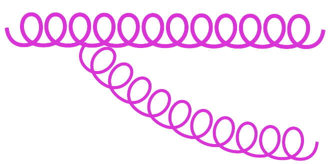
The Initial State: Color Glass Condensate & Saturation

$$Y = \ln \frac{p_0 + p_z}{p_0 - p_z}$$



ΔY

gluon radiation



p_z

$k_z = x\, p_z$

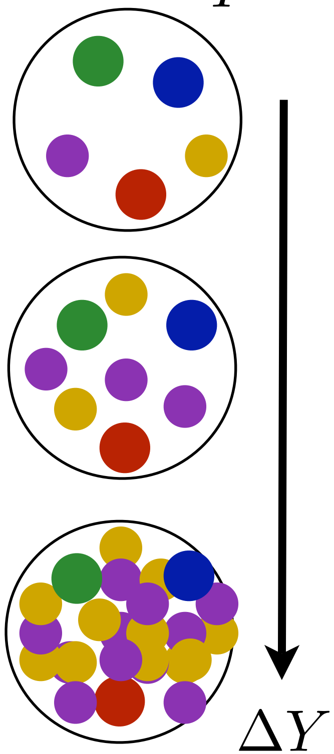
linear evolution (DGLAP, BFKL), dilute regime

$$\frac{\partial\, N_g}{\partial Y} \sim P\, N_g$$

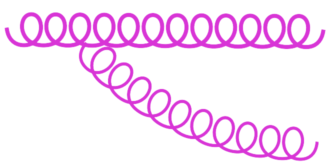
exponentially growing gluon densities

The Initial State: Color Glass Condensate & Saturation

$$Y = \ln \frac{p_0 + p_z}{p_0 - p_z}$$



gluon radiation



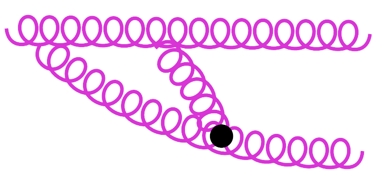
p_z

$k_z = x\, p_z$

linear evolution (DGLAP, BFKL), dilute regime

$$\frac{\partial N_g}{\partial Y} \sim P\, N_g$$

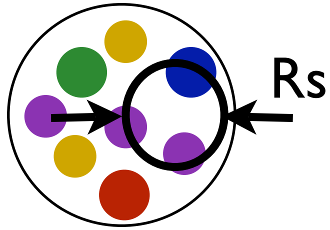
gluon recombination



Non-linear evolution (CGC), high density

$$\frac{\partial N_g}{\partial Y} \sim P\, N_g - R\, N_g^2$$

- At high energies (large rapidities, small-x), the hadron wavefunction reach saturation due to the growing importance of recombination processes



$$Q_s \sim \frac{1}{R_s} \qquad k_t < Q_s(Y)$$

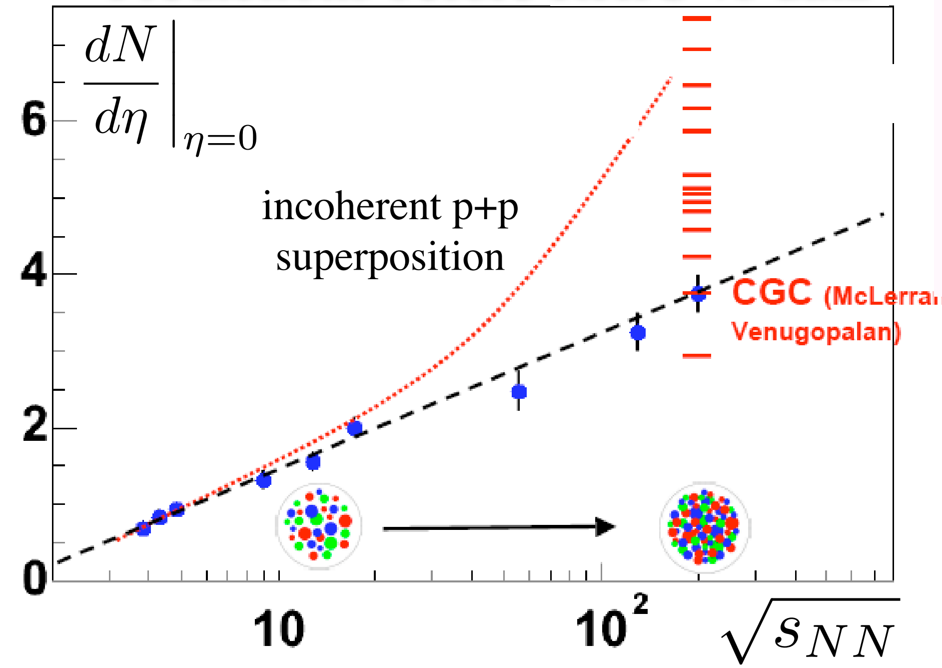
- Saturation is enhanced in nuclei (large # of gluons, even at low energies)

$$Q_{sA}^2 \sim A^{1/3}\, Q_{sp}^2 \quad \Rightarrow \quad A^{1/3} \sim 6 \quad \Rightarrow \quad Q_{sAu}^{2,\, RHIC} \sim 1 \div 2\, \mathrm{GeV}^2$$

Bulk properties of RHIC matter: Multiplicities

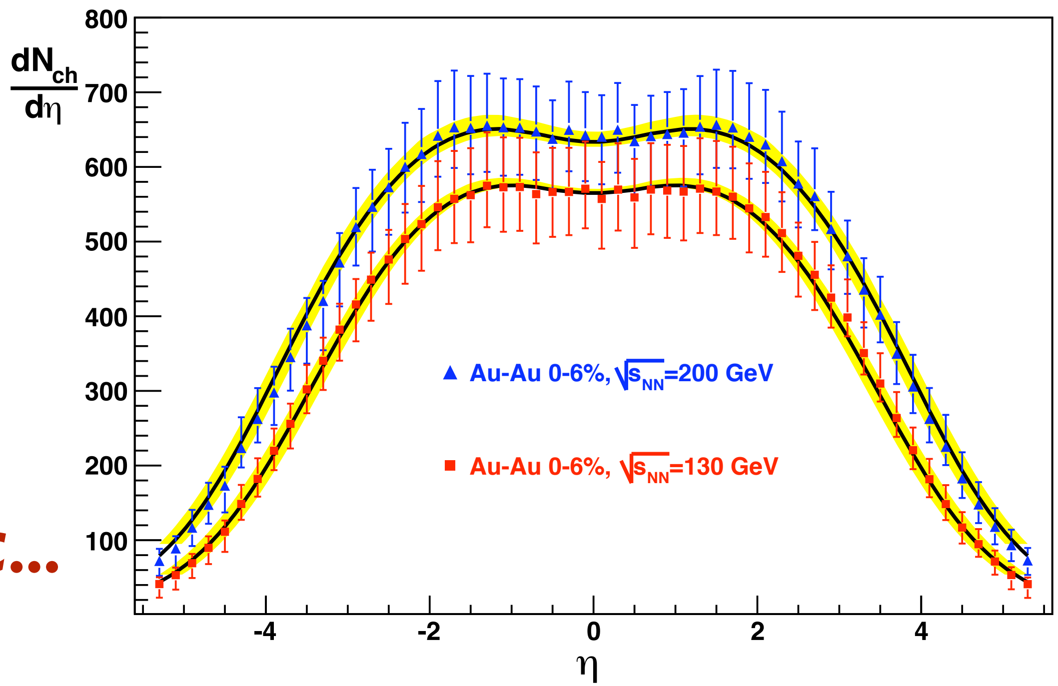
- One expects the total # of produced hadrons to be proportional to the # of partons in the wavefuncttin of colliding nuclei
- First surprise at RHIC: Total multiplicities came out a lot smaller than predicted by simple superpositions of proton-proton collisions:

Predictions before RHIC vs data



- Saturation explanation: The flux of colliding partons (mostly gluons) is reduced due to saturation effects

- CGC predictions account the energy rapidity, centrality of the multiplicities



… CGC has been discovered at RHIC…

The success of hydrodynamics at RHIC

⇒ Hydrodynamics is an effective theory that describes the long wavelength modes of the conserved charges of the system

energy-momentum conservation: $\partial_\mu T^{\mu\nu} = 0$

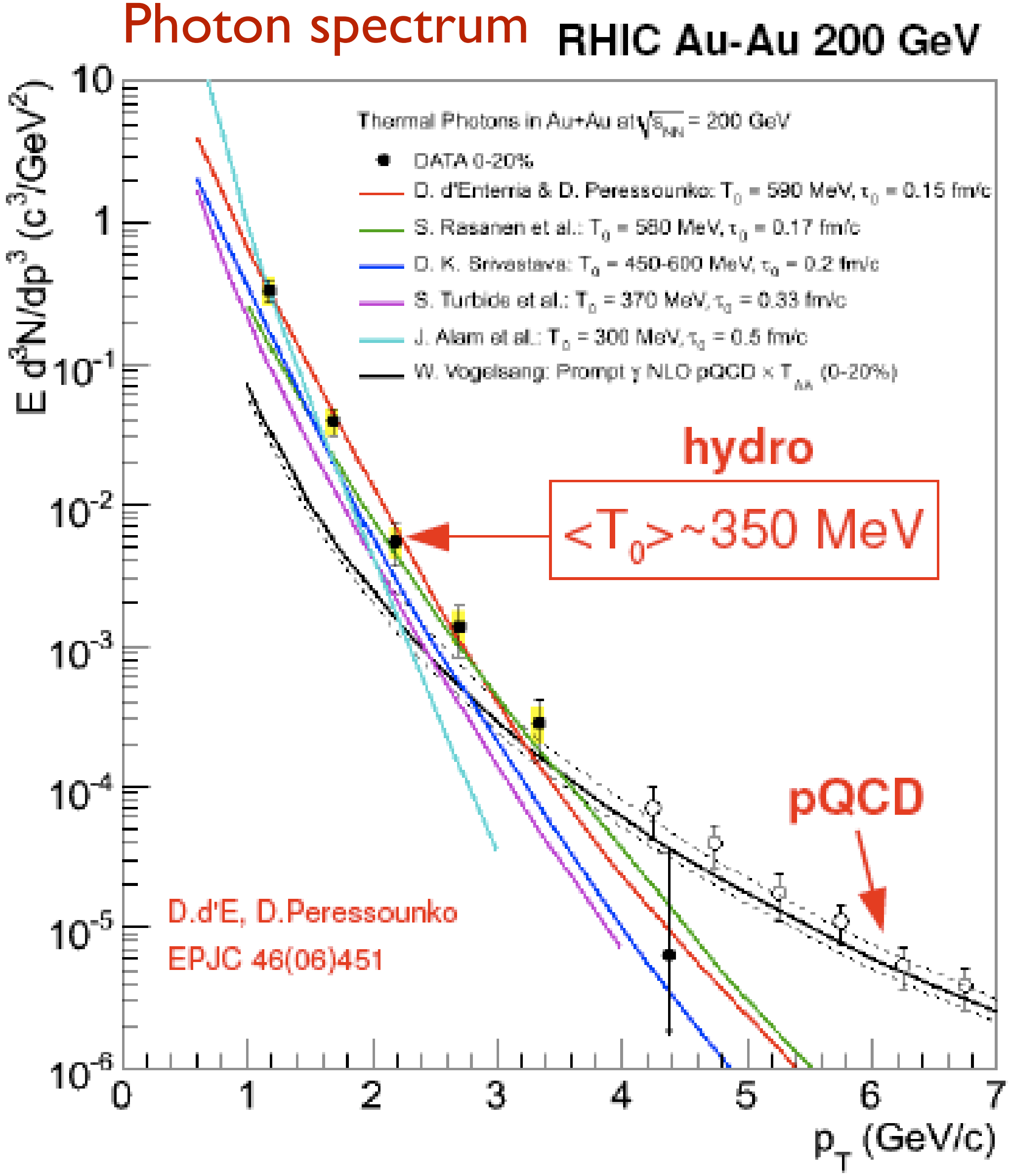
baryon number conservation: $\partial_\mu j^\mu_B = 0$

⇒ It requires local equilibrium and a small mean free path: $\lambda_{mfp} \sim (\sigma\, n)^{-1} \to 0$

$$T^{\mu\nu} = \underbrace{[\epsilon(p,T) + p]\, u^\mu u^\nu - p\, g^{\mu\nu}}_{\text{ideal fluid}} \underbrace{+ F(\nabla_\mu u^\nu; \eta; D \ldots)}_{\text{dissipative terms (viscosity...)}}$$

⇒ Ideal hydro describes a lot of RHIC data!!

Photon spectrum



Input for hydro evolution:

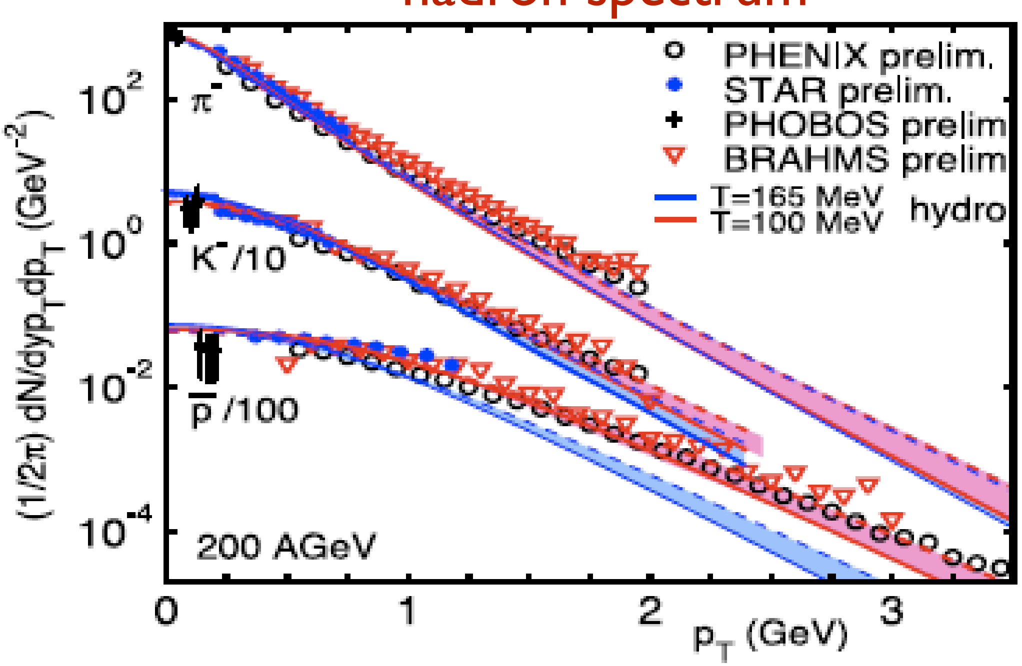
- QGP E.oS.
- short thermalization time:

$$\tau_{therm} \sim 0.6 \div 1\,\text{fm}$$

- initial energy density:

$$\epsilon_{\tau_{therm}} \sim 30\,\text{GeV/fm}^3$$

hadron spectrum



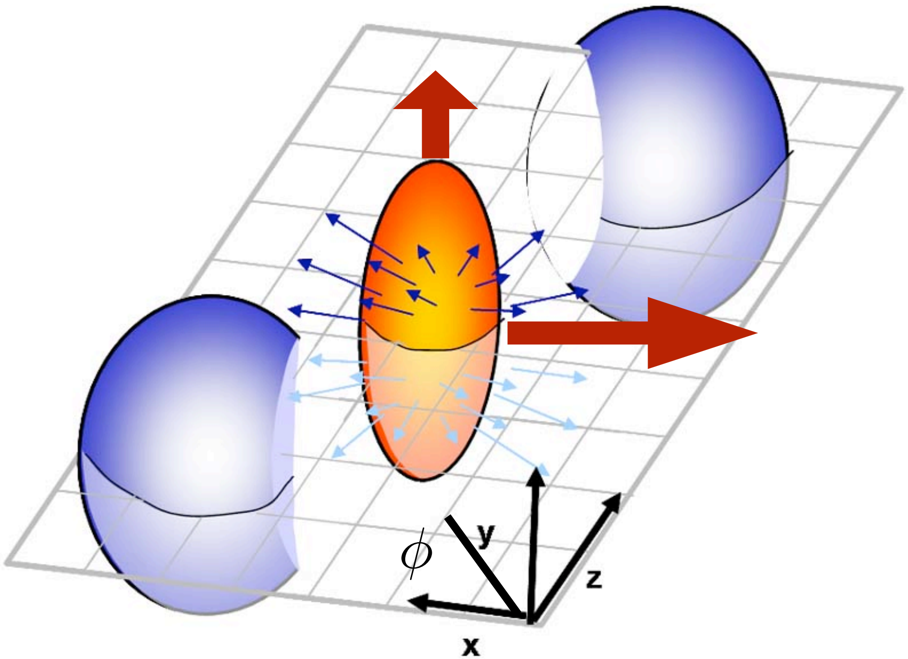
RHIC matter flows: Elliptic (and radial) flow

⇒ The initial fireball produced in non-central collisions is highly anisotropic

$$\dot{u}^\mu = \frac{\nabla^\mu p}{\epsilon + p}$$

⇒ If the system behaves like a fluid, the initial spatial anysotropy is mapped onto the observed hadron spectra

$$\frac{d\,N^h}{d^2 p_t\, d\phi} \propto 1 + 2\,\mathbf{v_2}(\mathbf{p_t})\,\cos(2\phi) + \ldots$$



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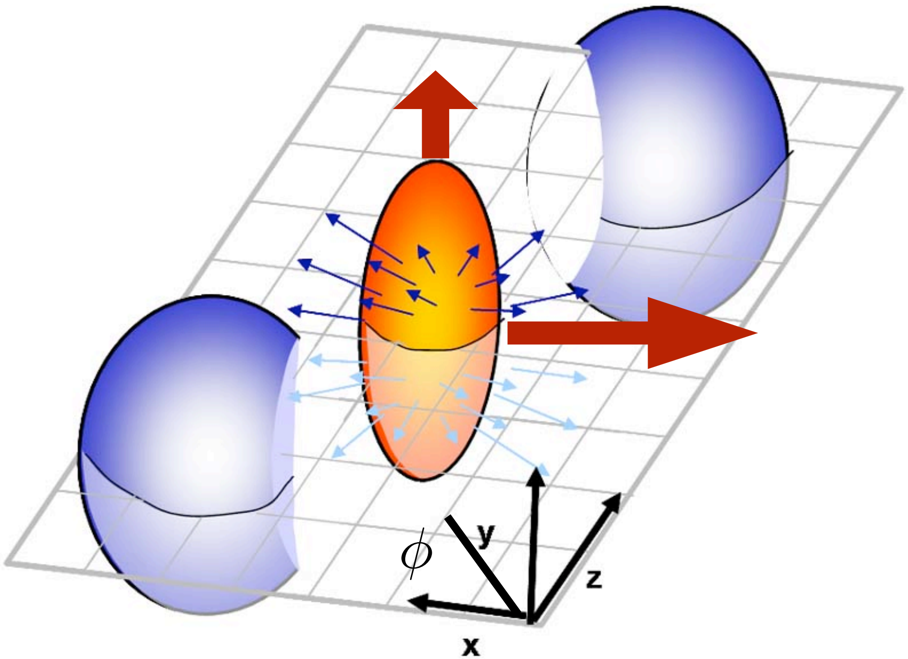
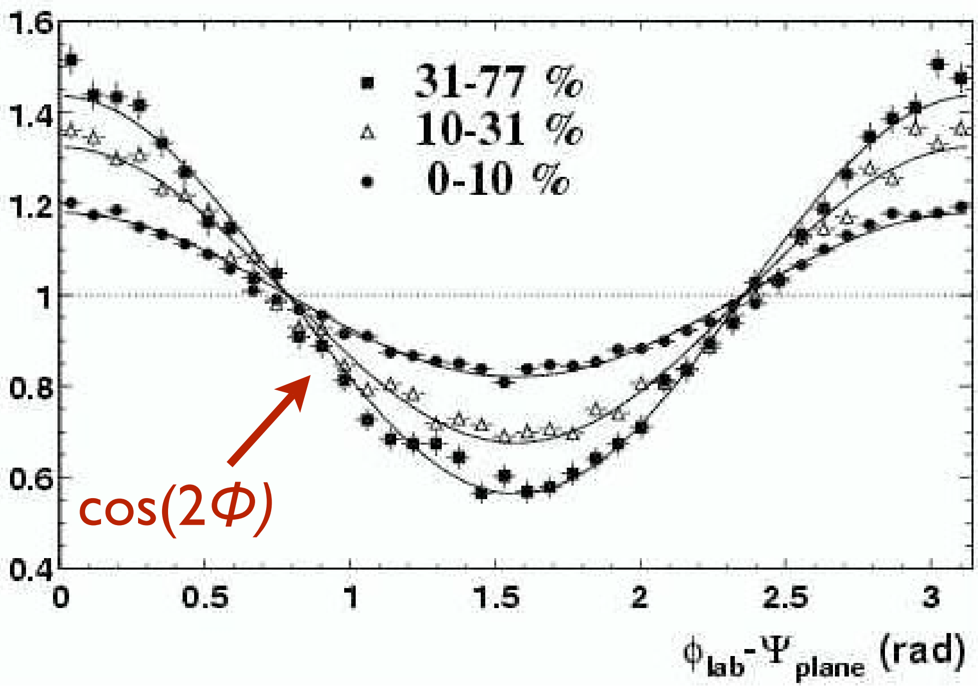
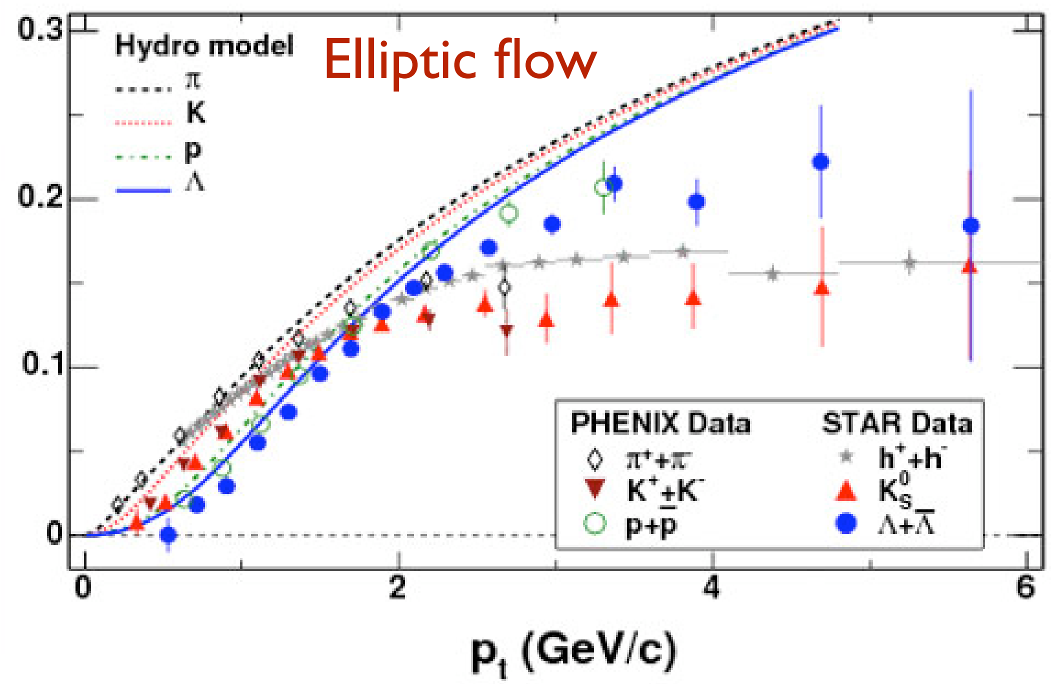


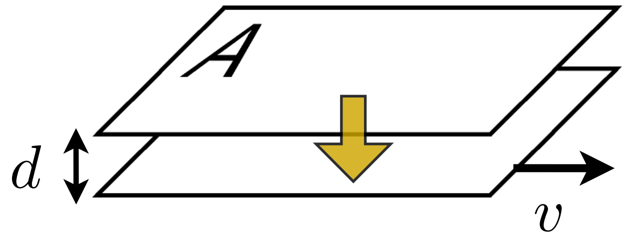




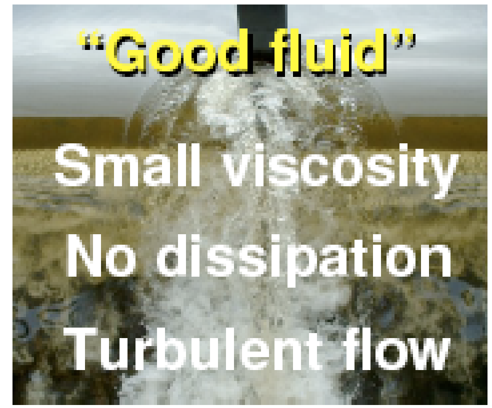
The most perfect fluid?

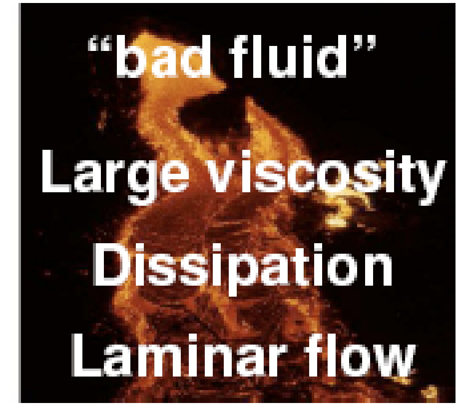
⇒ Viscosity ~ "internal friction of a fluid"

η ~ 1/fluidity.



$$F = \eta \, \frac{A}{d} \, v$$



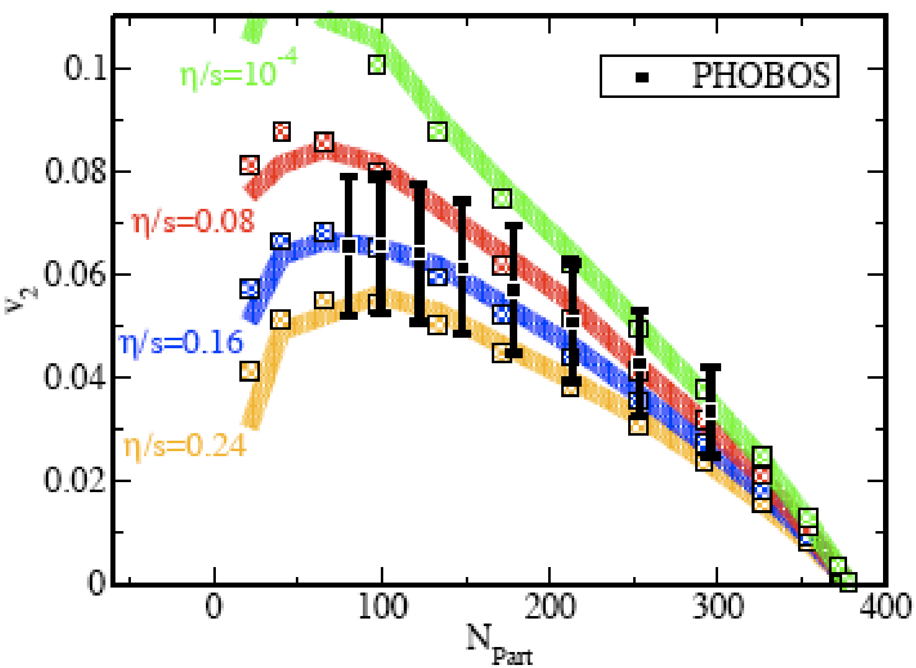


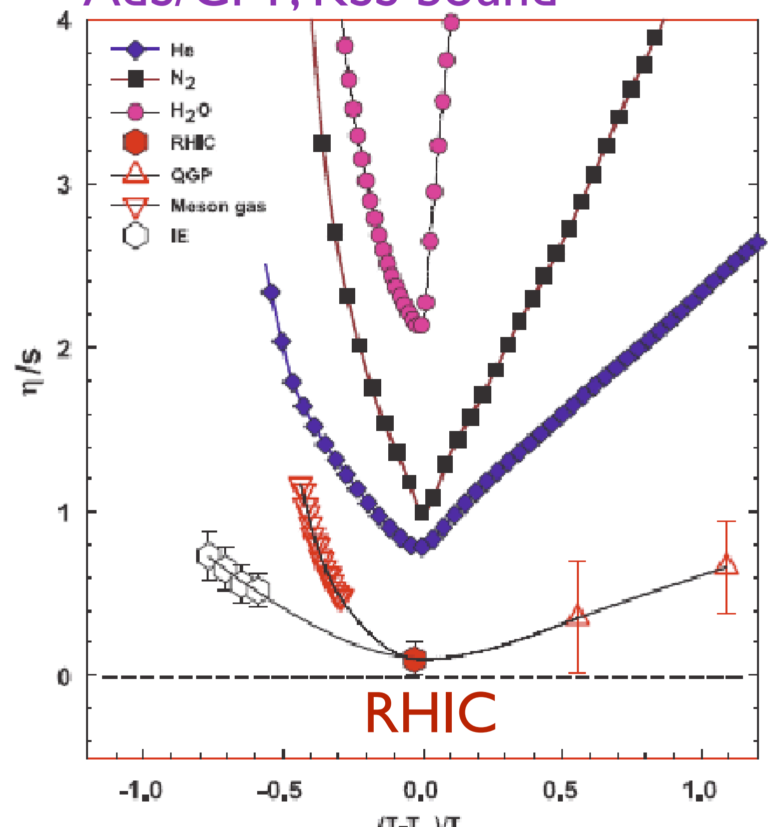
⇒ Minimum viscosity/entropy ratio: $\frac{\eta}{s} \geq \frac{1}{4\pi} \frac{\hbar}{k_B}$

Uncertainty principle

AdS/CFT; KSS bound

⇒ QGP (from hydrodynamics): $\frac{\eta}{s} \sim \frac{1 \div 6}{4\pi} \frac{\hbar}{k_B}$

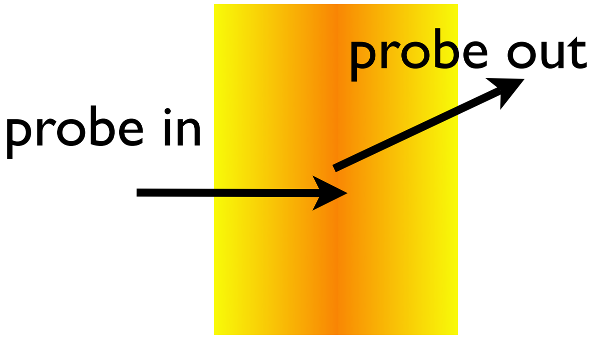




Hard Tomographic Probes:

⇒ Particles with a large momentum (mass) scale M: jets, γ , $Q\bar{Q}$ ...

- Well controlled theoretically (pQCD) and experimentally
- Produced at early times t~1/M in (rare) hard collisions
- The modification tells us about the medium properties



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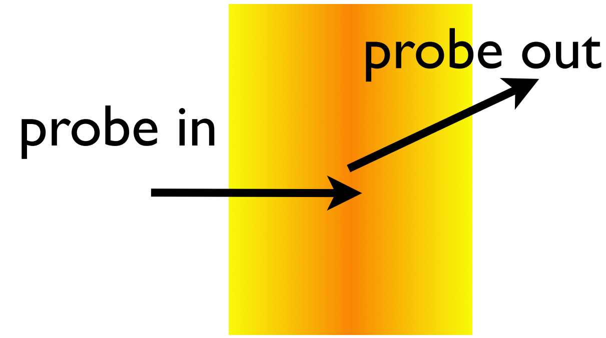
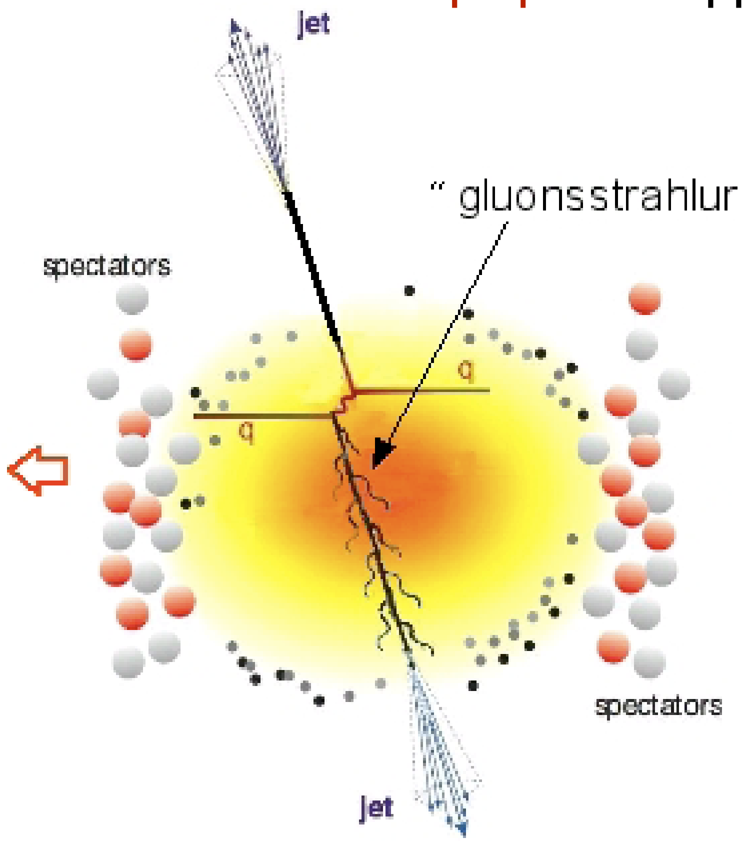
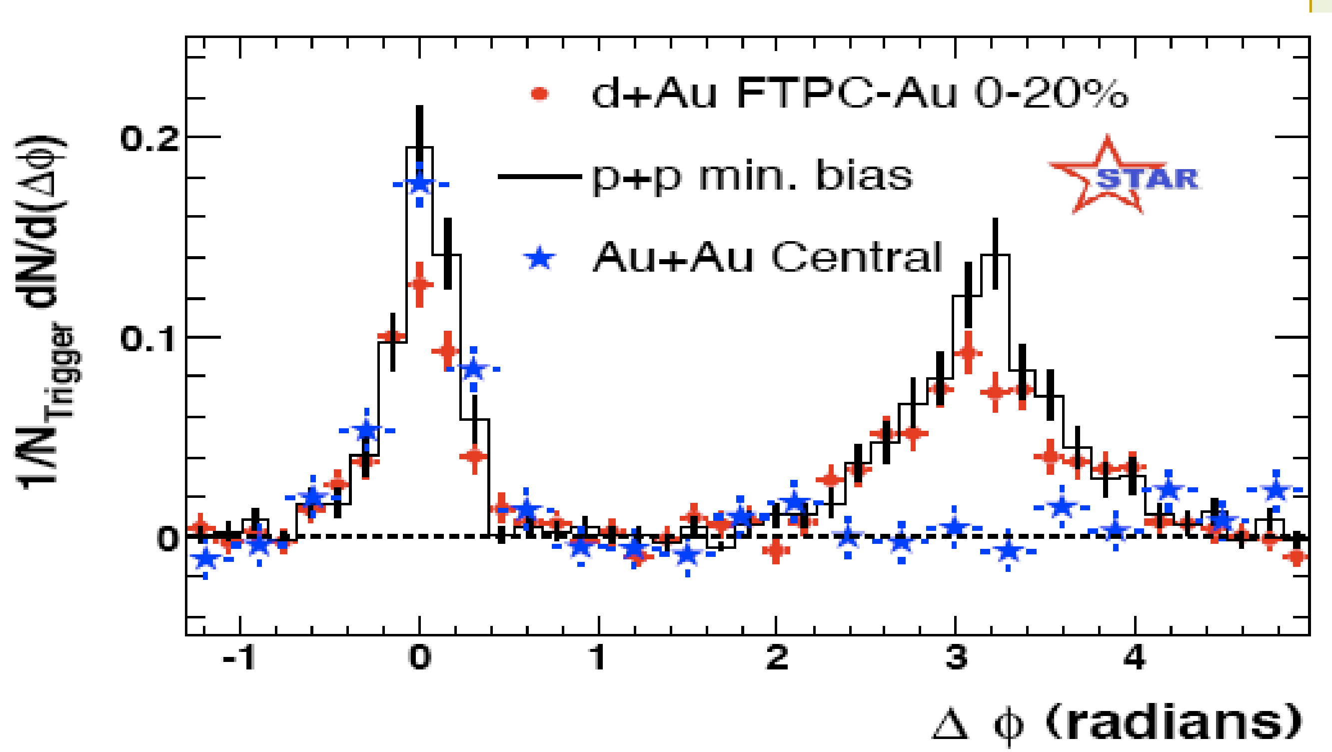


- RHIC matter is opaque: Disappearance of back to back correlations in Au-Au collisons:





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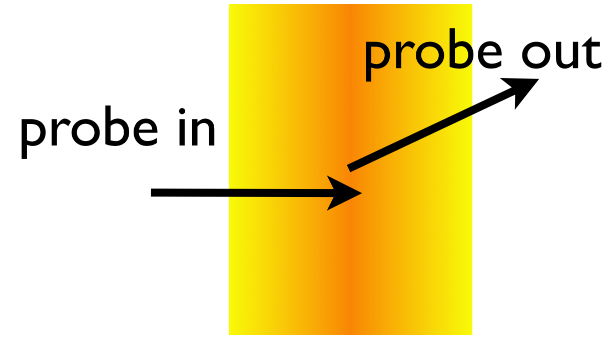


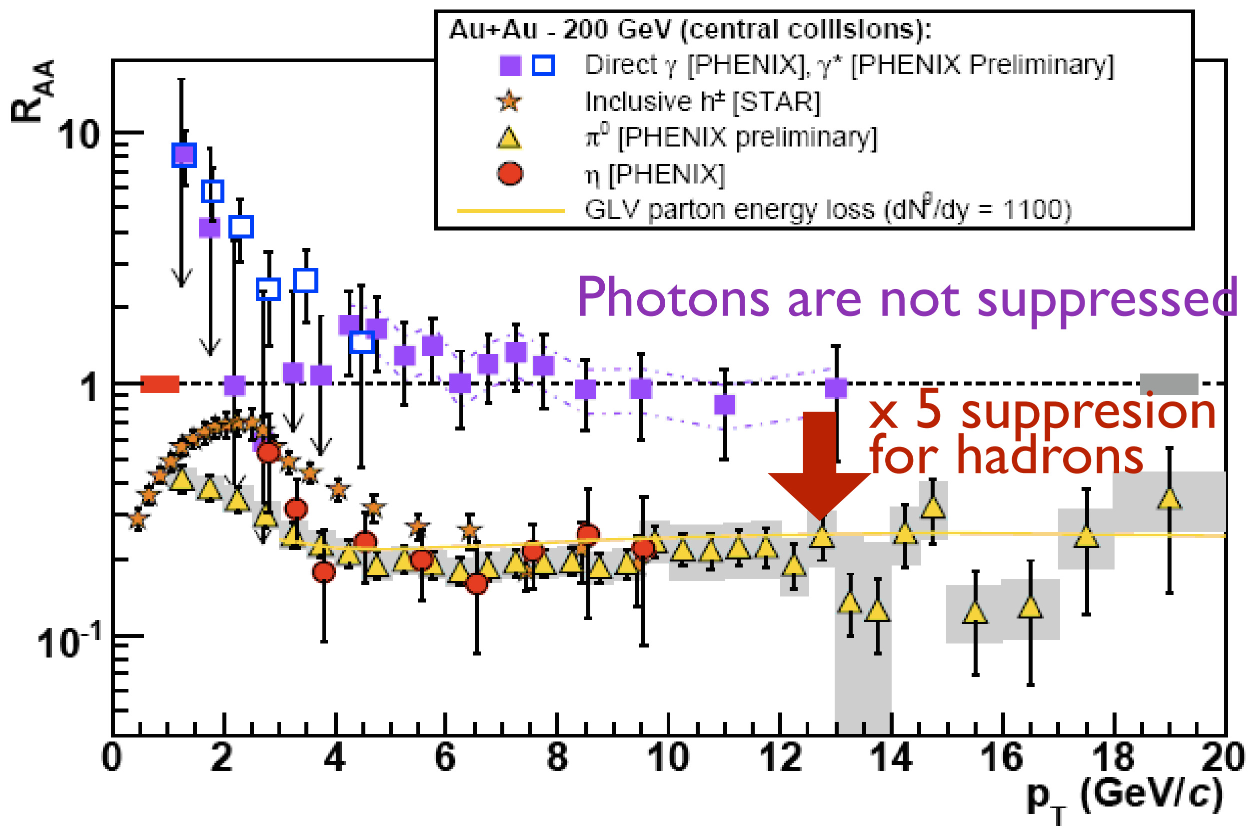
- RHIC matter is opaque: Strong suppression of high-pt hadrons

Parton energy-loss

$$\langle \Delta E \rangle \propto \alpha_s C_R \langle \hat{\mathbf{q}} \rangle L^2$$

Transport coefficient

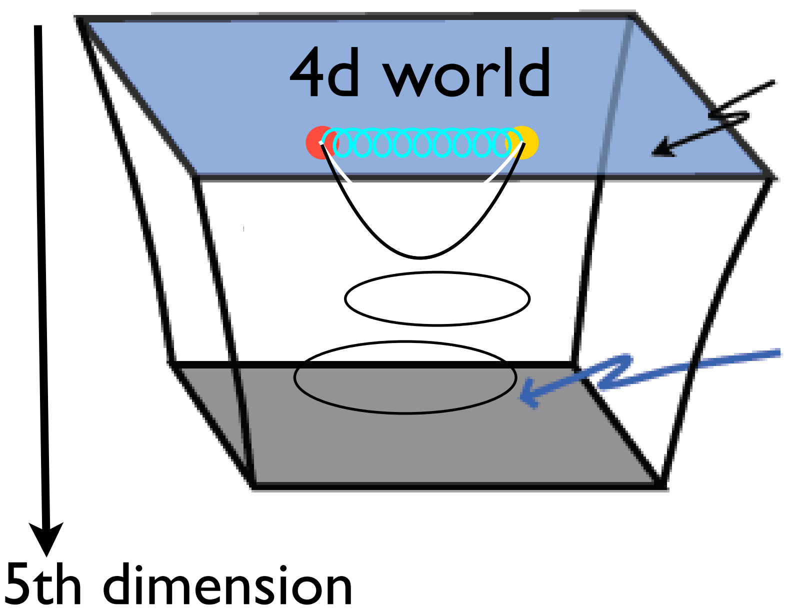
$$\hat{q} \sim 4 \frac{\mathrm{GeV}^2}{\mathrm{fm}}$$



The String Connection (or the weird couple)

- So RHIC matter behaves like a strongly interacting system (perfect fluid, jet quenching..)
- So we need a formalism that allows to study strongly coupled systems in real-time formalism (Lattice QCD operates in imaginary time)

The Anti de Sitter / Conformal Field Theory Correspondance (AdS/CFT)



Weakly coupled supergravity in $AdS_5 \times S_5$ space ⟷ N=4 SYM in 4d

$$\lambda = g_s^2 N_c \to \infty$$

$$N_c \to \infty$$

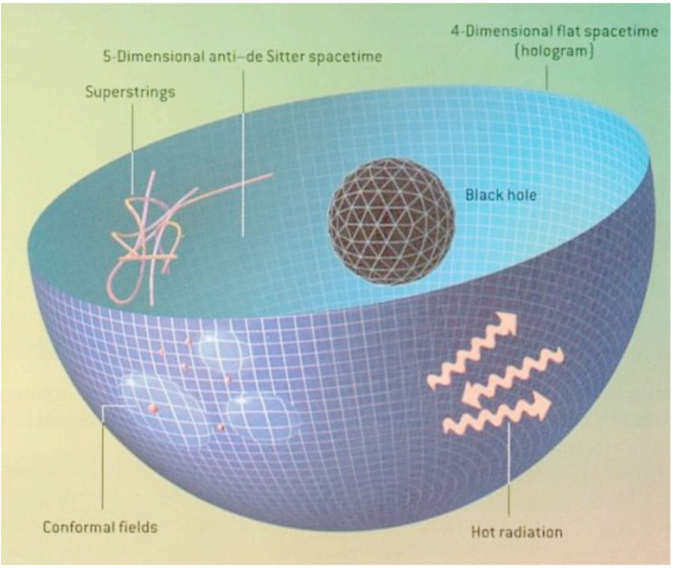
Black brane along the fifth dimension ⟷ Finite-T system

$$T = \frac{1}{\pi z_h}$$

Caveats: N=4 SYM is conformal. It is supersymmetric. It includes scalar and fermions. It has no charges in the fundamental representation (quarks)....

Some applications of the correspondence to HIC

- KSS viscosity bound (Kovtun, Starinets and Son) $\frac{\eta}{s} \geq \frac{1}{4\,\pi}\,\frac{\hbar}{k_B}$



CFT entropy = BH entropy: $s_{BH} = \frac{A_{BH}}{4\,\pi}$

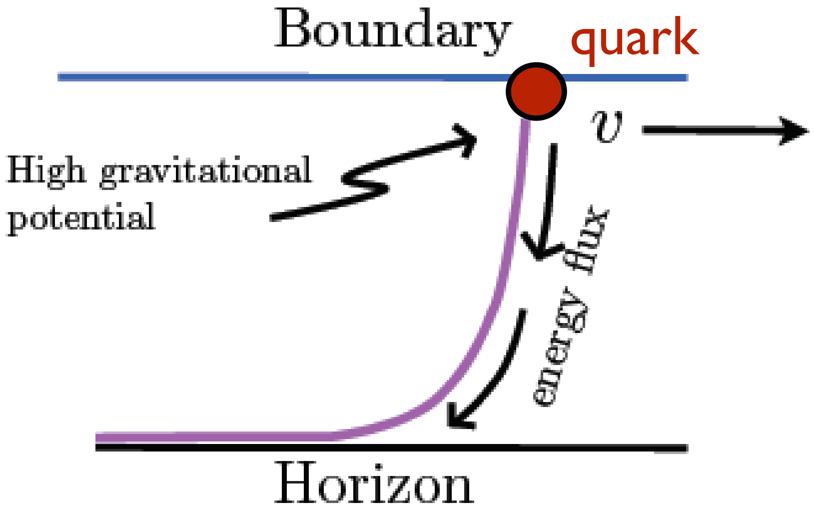
CFT viscosity $\eta = \lim_{w\to 0} \frac{1}{2w} \int dx\,dt \langle [T_{xy}(t,x), T_{x,y}(0,0)] \rangle$

$$\sim \sigma^{graviton}(0) = \frac{A_{BH}}{16\,\pi\,G}$$

$\Downarrow$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- Calculation of jet quenching (heavy quark difussion) and DIS on plasma:



AdS/CFT calculations yield a large jet quenching, compatible with the value extracted empirically

$$\hat{q} \sim 4\,\frac{\mathrm{GeV}^2}{\mathrm{fm}}$$

...Although there is some numerology involved here...

• Other proposed signatures of QGP formation:

-Enhancement of thermal photons and dileptons from black-body radiation
-Melting of heavy quark bound states (J/Ψ, Ψ ‘, γ..)
-Enhancement of strange production...

• Summary: Great progress achieved over the last 10 years in our understanding of the QCD phases. RHIC has delivered evidence for the formation of a strongly interacting, perfect-fluid-like Quark Gluon Plasma.

• Outlook: Many open questions: Dynamics of thermalization, microscopic composition of QGP around Tc, development of full viscous hydrodynamics, coupling of soft (hydro) modes and fast (jets), sharpening our understanding of the AdS/CFT correspondence, species dependence of the suppression, jet studies ....

• The answers will (most likely) come from a combination of experimental results (LHC, FAIR), theoretical developments (in progress) and improvements of Lattice-QCD numerical simulations

Back up slides

Beyond LL approximation: Running coupling corrections

(Kovchegov-Weigert, Balitsky, Gardi et al 06, Albacete-Kovchegov)

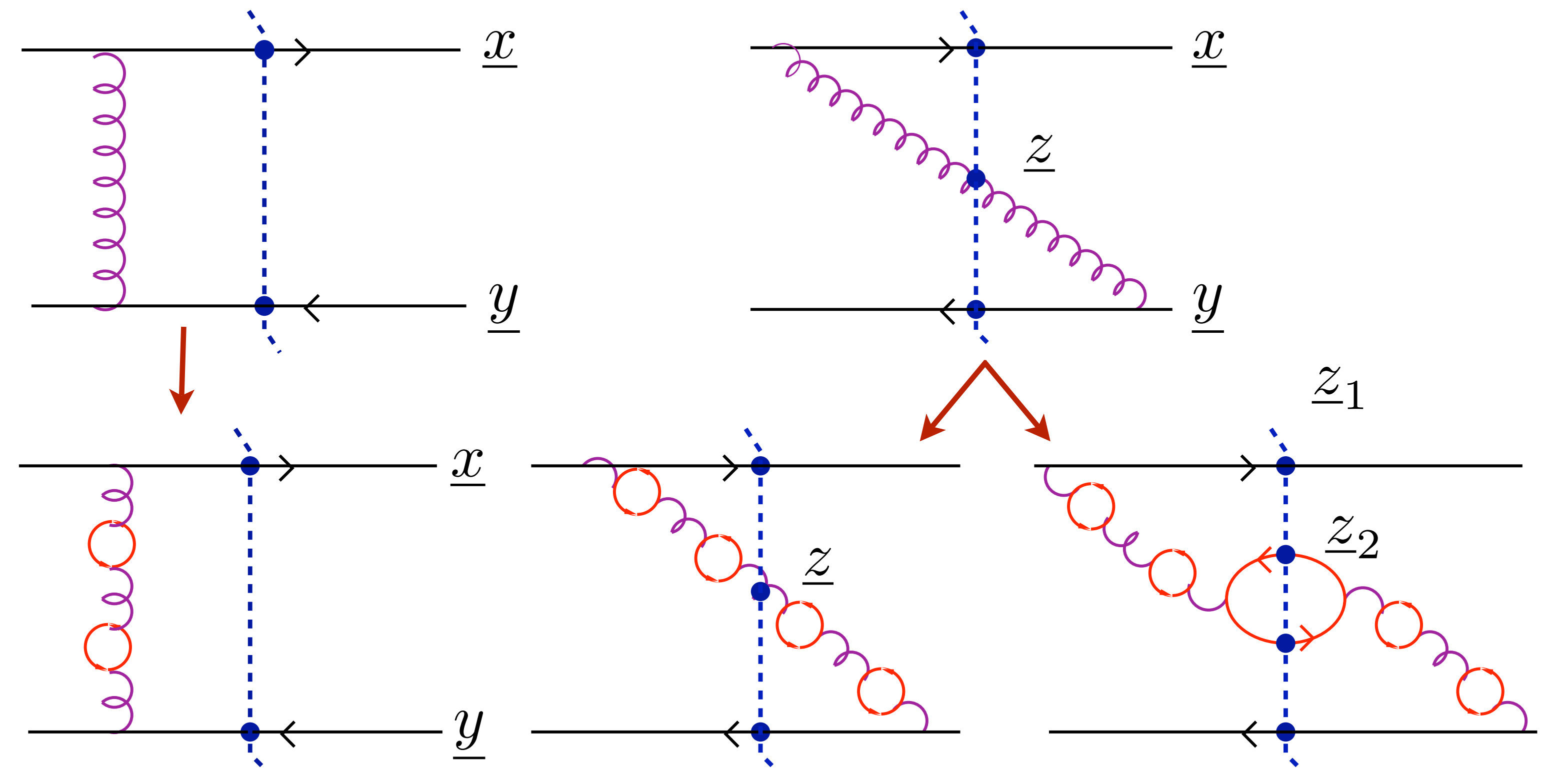
Strategy: resummation of quark loops to all orders, plus $N_f \longrightarrow -6\pi\beta$

⇒ Leading log
(fixed coupling)

⇒ All orders in $\alpha_s N_f$

$N_f \longrightarrow -6\pi\beta$

(running coupling)



Running coupling corrections (Kovchegov-Weigert, Balitsky, Weigert et al 07)

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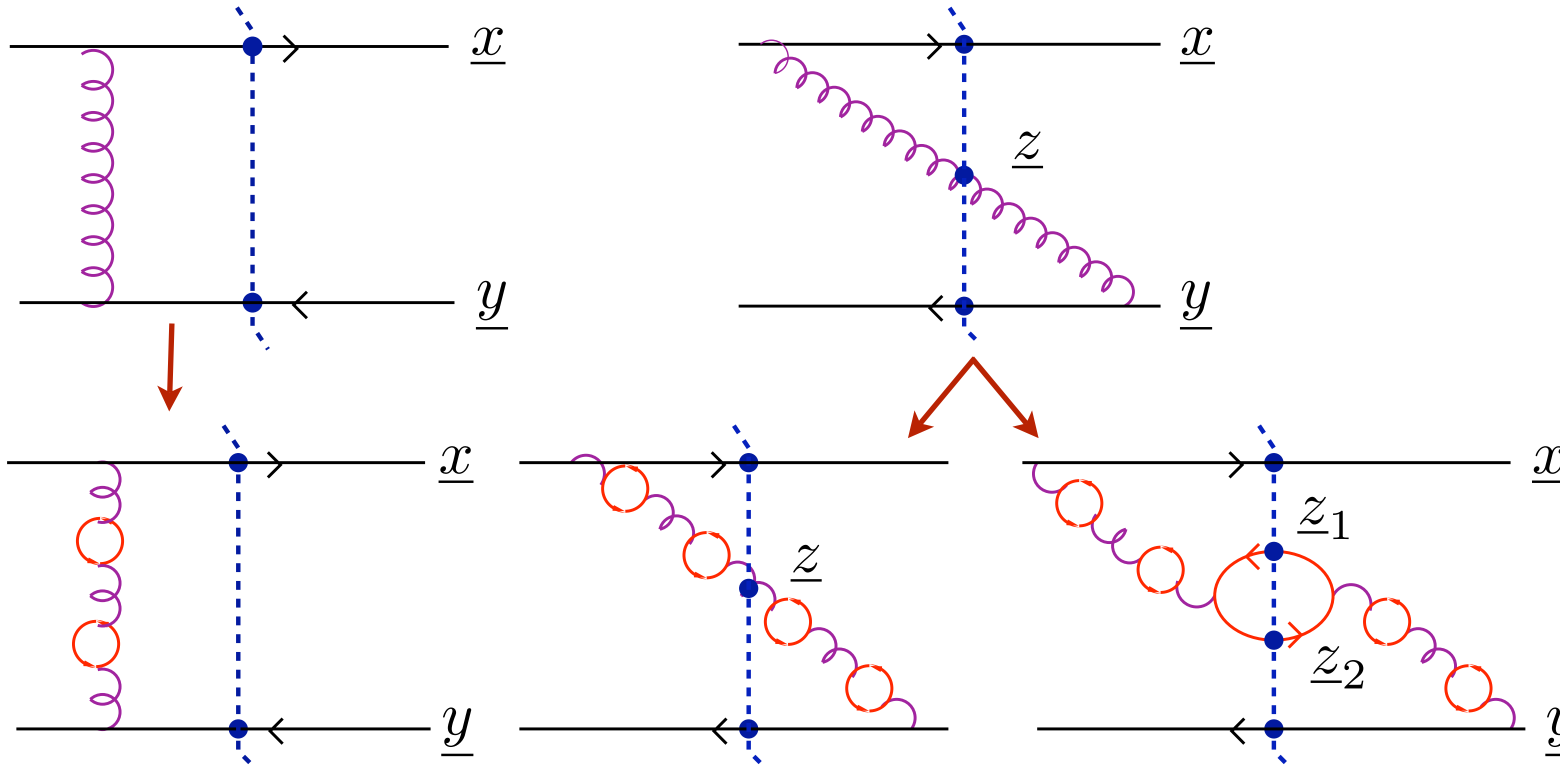


⇒ All orders in $\alpha_s N_f$

$N_f \longrightarrow -6\pi\beta$

(running coupling)

$$\alpha_\mu \ln\left(\frac{1}{x}\right)\left[1+\sum_n c_n\left(\alpha_\mu \frac{N_f}{6\pi}\ln\frac{q^2}{\mu^2}\right)^n\right]$$

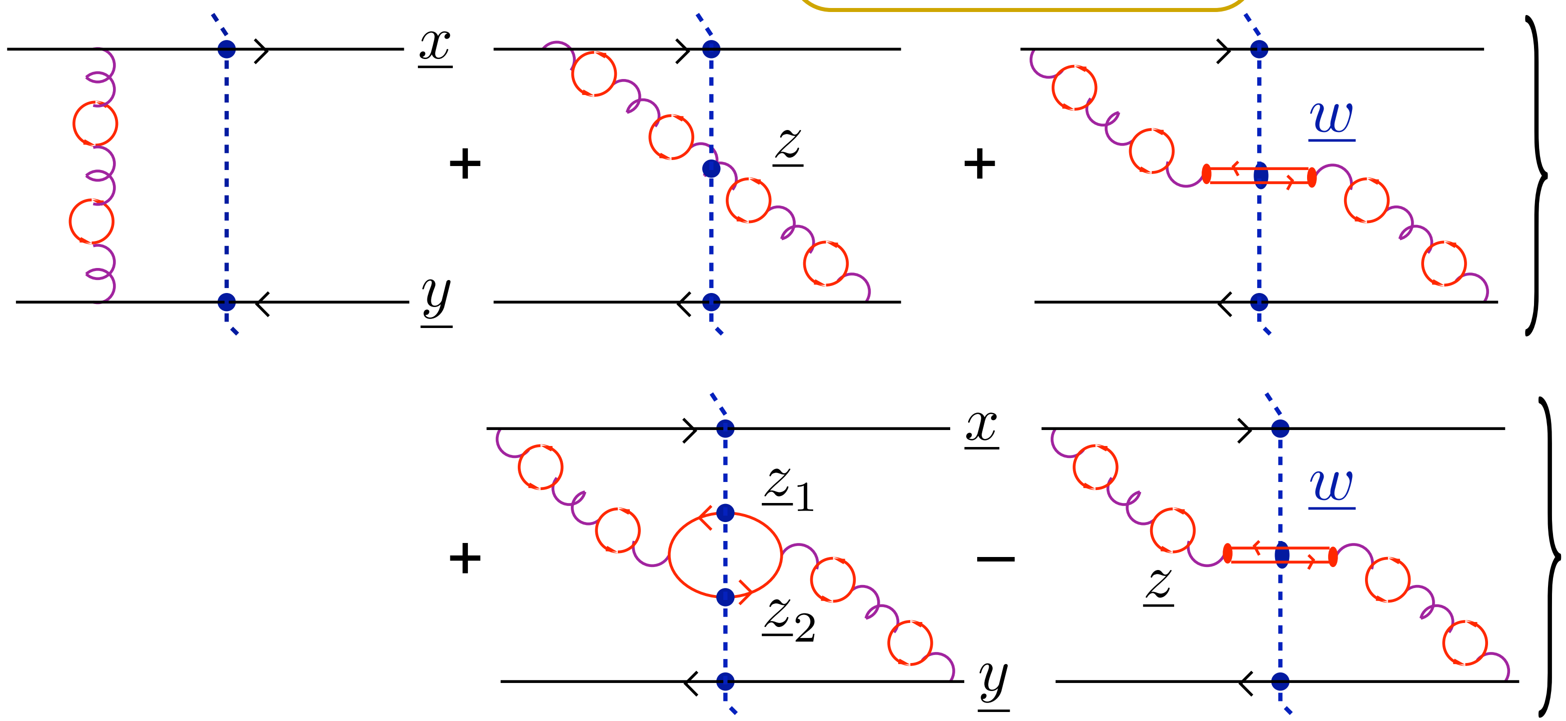
Resumming the geometric series
$N_f \rightarrow -6\pi\beta_2$

$$\Rightarrow \ln(1/x)\,\frac{\alpha_\mu}{1+\alpha_\mu\beta_2\ln\left(\frac{C\,q^2}{\mu^2}\right)}$$

Complete in $\alpha_s N_f$ Evolution

JLA and Y. Kovchegov PRD75 125021

$$\frac{\partial S}{\partial Y} = \mathcal{R}\,[S] - \mathcal{S}\,[S]$$



$\mathcal{R}[S]$
UV-divergent terms that contribute to the running of the coupling

$\mathcal{S}[S]$
Conformal, non running coupling terms. Neglected in previous calculations

⇒ ***Running*** term: $\mathcal{R}\,[S] = \int d^2 z\, \tilde{K}(\underline{r}, \underline{r}_1, \underline{r}_2)\, \left[S(\underline{x}, \underline{z})S(\underline{z}, \underline{y}) - S(\underline{x}, \underline{y})\right]$

⇒ ***Subtraction*** term: $\mathcal{S}\,[S] = \int d^2 z_1 d^2 z_2\, K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2)\, \left[S(\underline{x}, \underline{w})S(\underline{w}, \underline{y}) - S(\underline{x}, \underline{z}_1)S(\underline{z}_2, \underline{y})\right]$

⇒ Running coupling comes in a "triumvirate": $K \sim \dfrac{\alpha_s(R_1)\,\alpha_s(R_2)}{\alpha_s(R_3)}$

Fixed vs Running

⇒ The running of the coupling reduces the speed of the evolution down to values compatible with experimental data (JLA PRL 99 262301 (07)):

$$\frac{\partial S}{\partial Y} = \mathcal{R}\,[S] - \mathcal{S}\,[S]$$

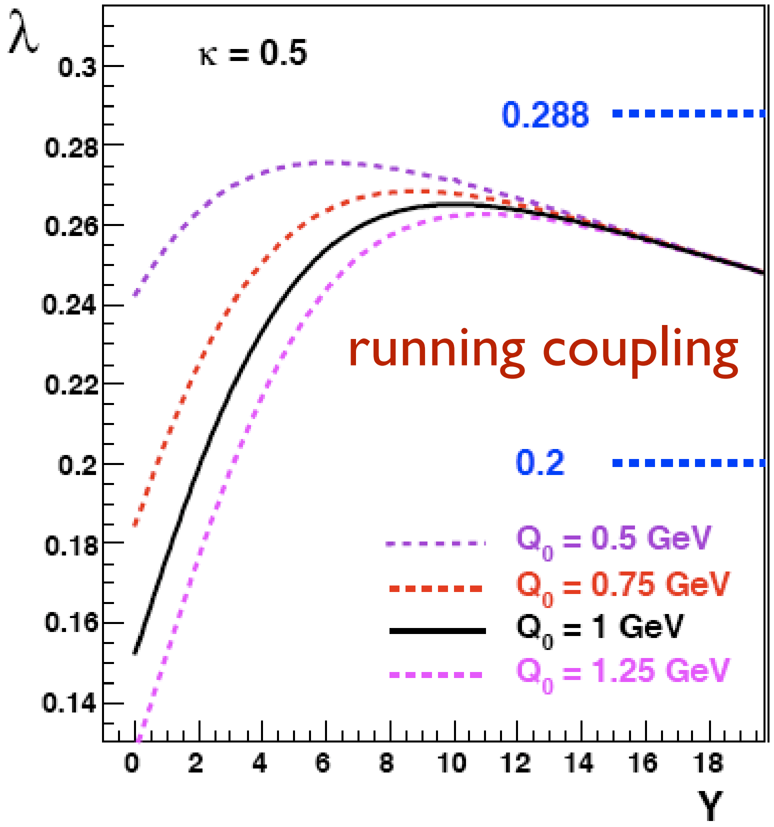
$$\lambda = \frac{d \ln Q_s^2(Y)}{dY}$$

LL evolution:

$$\lambda^{LL} \approx 4.8\,\alpha_s$$

DIS data:

$$\lambda^{DIS} \approx 0.288$$



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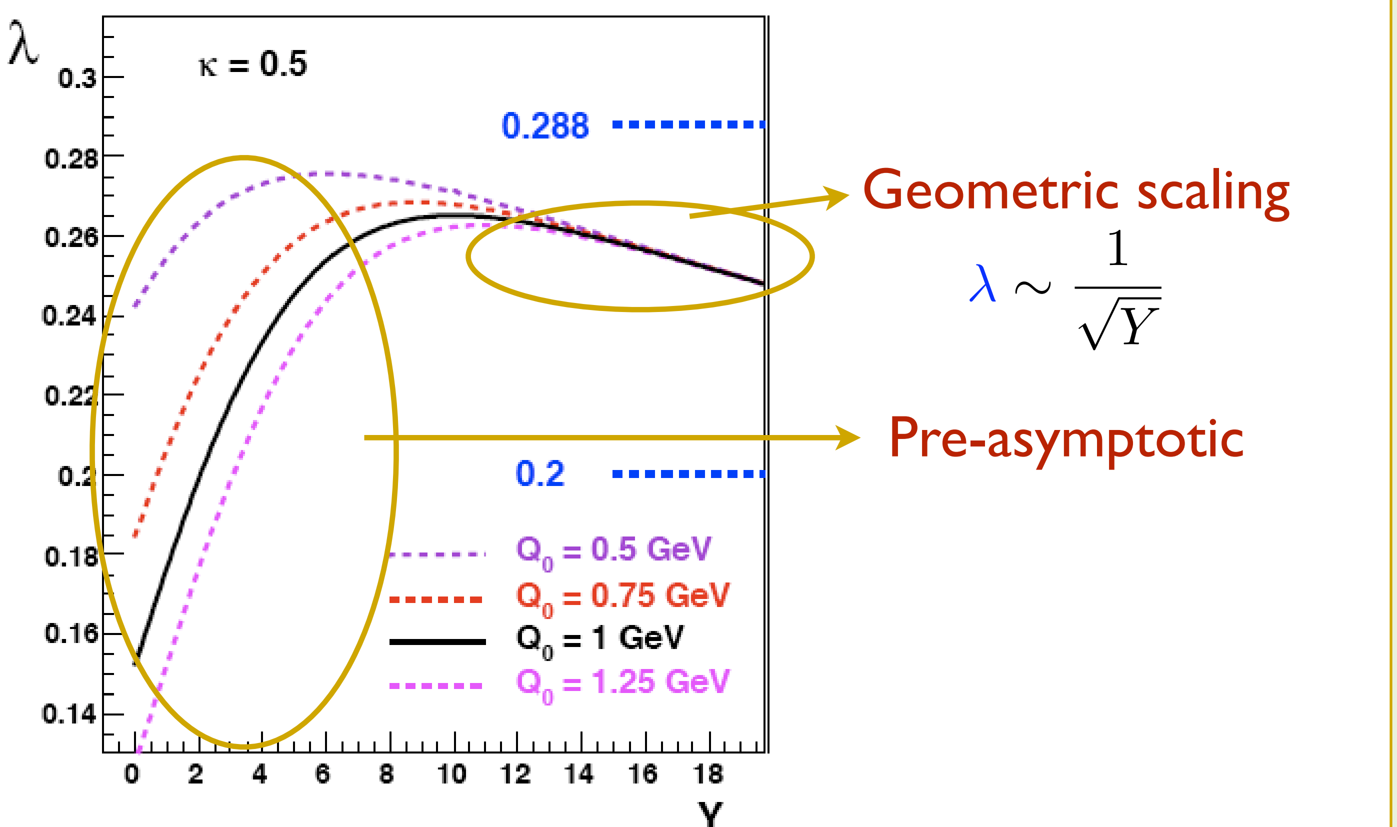


$$\lambda \sim \frac{1}{\sqrt{Y}}$$

⇒ Geometric scaling persists, despite conformal symmetry being broken

⇒ UNIVERSALITY

$$\frac{Q^2_{s\,A}(Y)}{Q^2_{s\,B}(Y)} \to 1 \quad \text{for } Y \to \infty$$

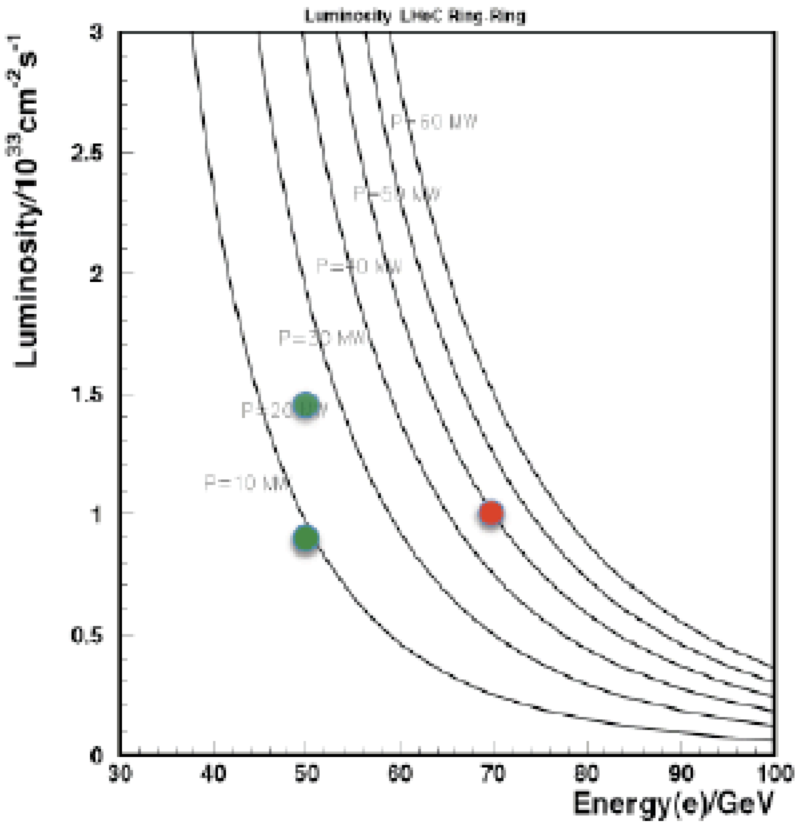
	EIC	LHeC
Community	US-nuclear + BNL & JLAB have declared project as key for future	European particle physicist + contacts with nuclear community
Parameters	3-20 GeV e 25-250 GeV p, ½ for A e,P polarized Lumi 10^{32}-10^{34}	20-120 GeV e 7 TeV p, ½ for A No P,A polarization Lumi 10^{32}-10^{33}
Physics	Spin 3D proton/nuclear structure pdfs	TeV scale, BSM Small-x physics pdfs
Cost	150 – 1400 M$ (US accounting)	TBD
Time Scale	≥2020 (necessary ?) Wide range of cost/scope, push for early staged project ?	Depends on LHC results Scenarios TBD

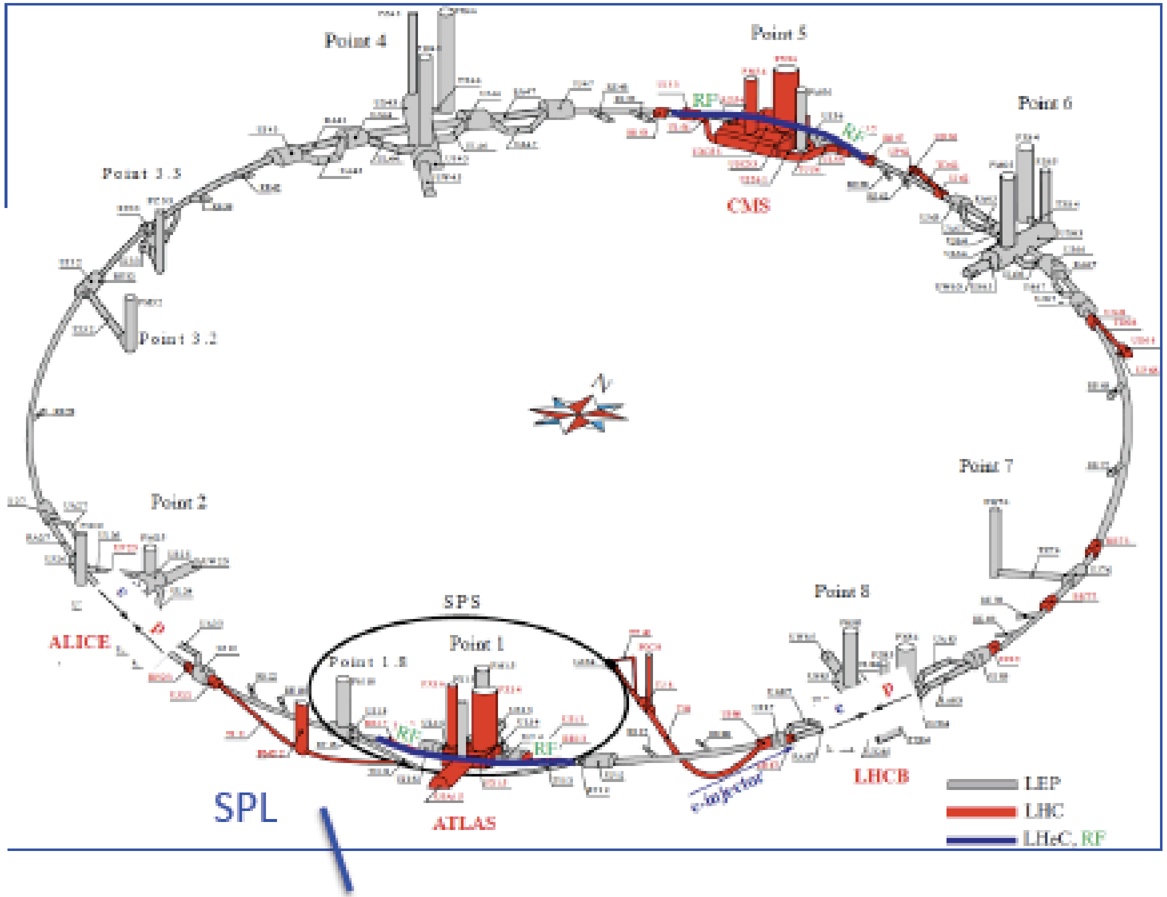
LHeC option 1

Ring – Ring Design tentative

$$L = \frac{N_p \gamma}{4\pi e \varepsilon_{pn}} \cdot \frac{I_e}{\sqrt{\beta_{px}\beta_{py}}} = 8.3\,10^{32} \cdot \frac{I_e}{50mA} \frac{m}{\sqrt{\beta_{px}\beta_{pn}}} cm^{-2}s^{-1}$$

$$I_e = 0.35mA \cdot \frac{P}{MW} \cdot \left(\frac{100GeV}{E_e}\right)^4$$





$$2 \cdot 10^{33} cm^{-2}s^{-1}, 5y : 100\, fb^{-1} \ldots 50 \ldots 80 GeV$$

F.Willeke, 70GeV * 7TeV, 50MW [JINST 2006]

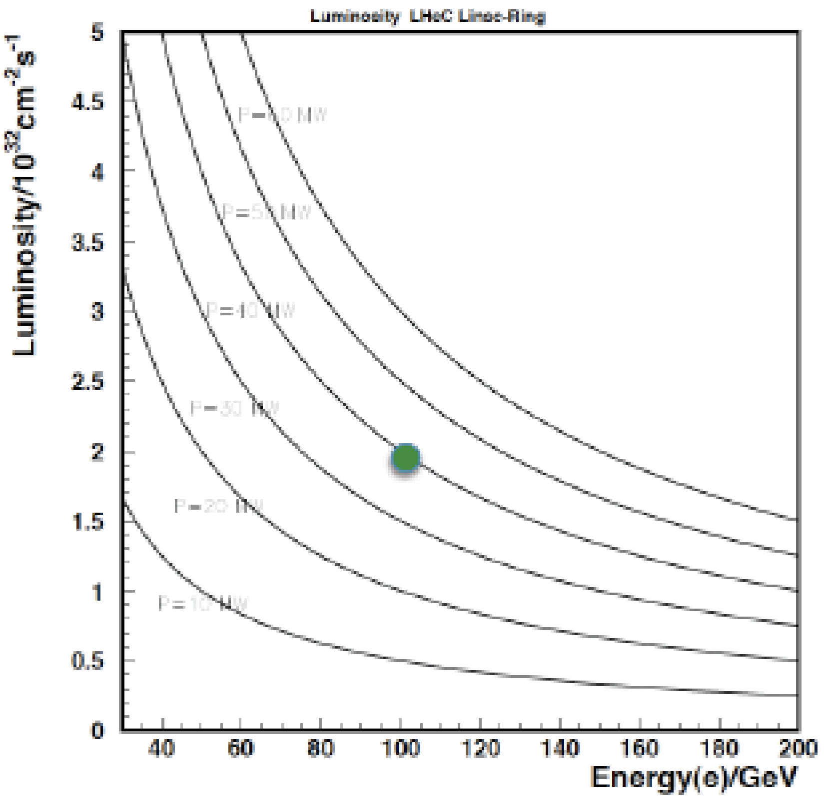
B.Holzer, A.Kling et al, Divonne08,ECFA08

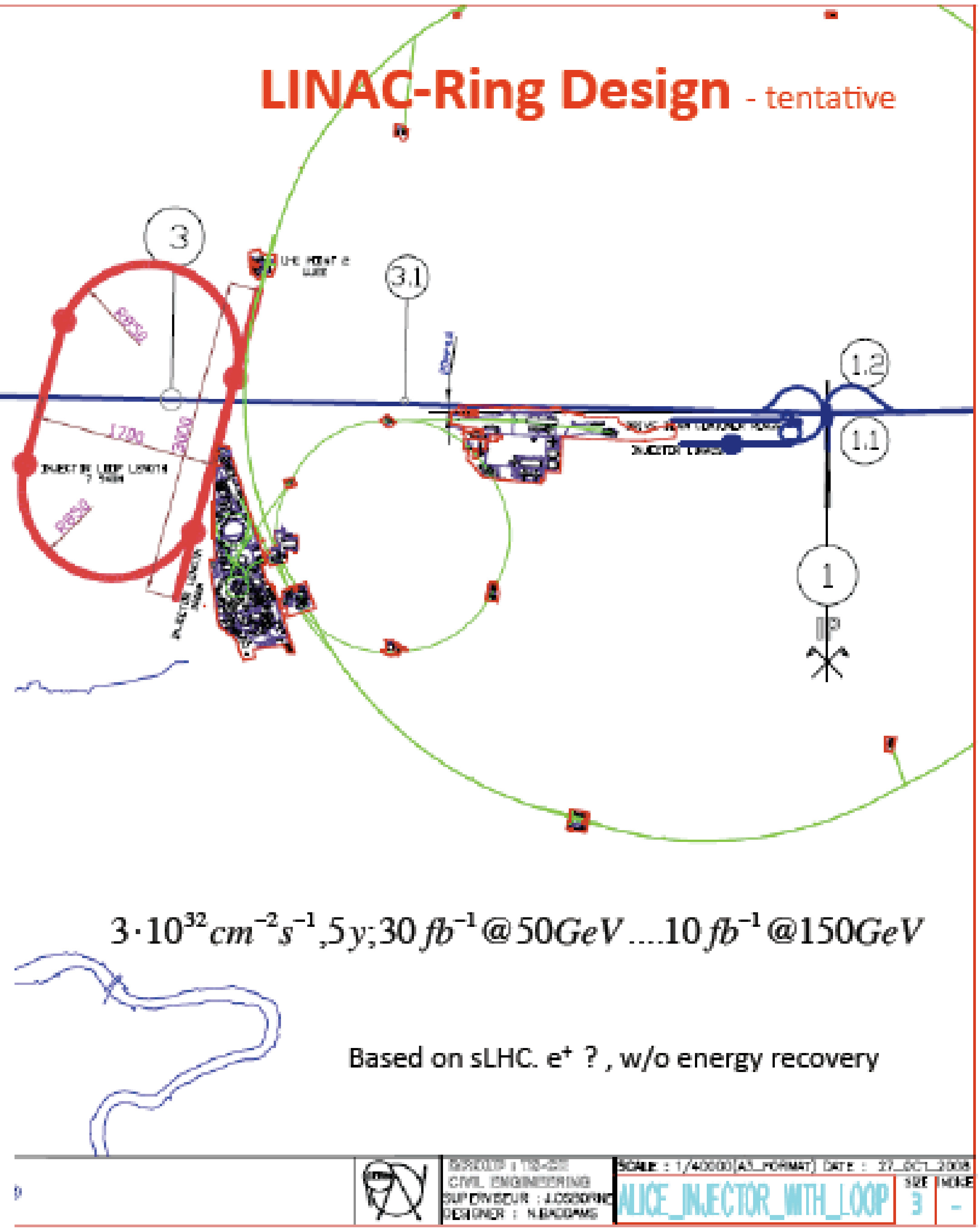
Subject to LHC, power, tuneshifts etc. – 100 times HERA luminosity

LHeC option 2

$$L = \frac{N_p \gamma}{4\pi e \varepsilon_{pn} \beta^*} \cdot \frac{P}{E_e} = 5 \cdot 10^{32} \cdot \frac{P/MW}{E_e/GeV} cm^{-2}s^{-1}$$

LINAC-Ring Design - tentative





$3 \cdot 10^{32} cm^{-2}s^{-1}, 5y; 30\, fb^{-1} @ 50GeV \ldots 10\, fb^{-1} @ 150GeV$

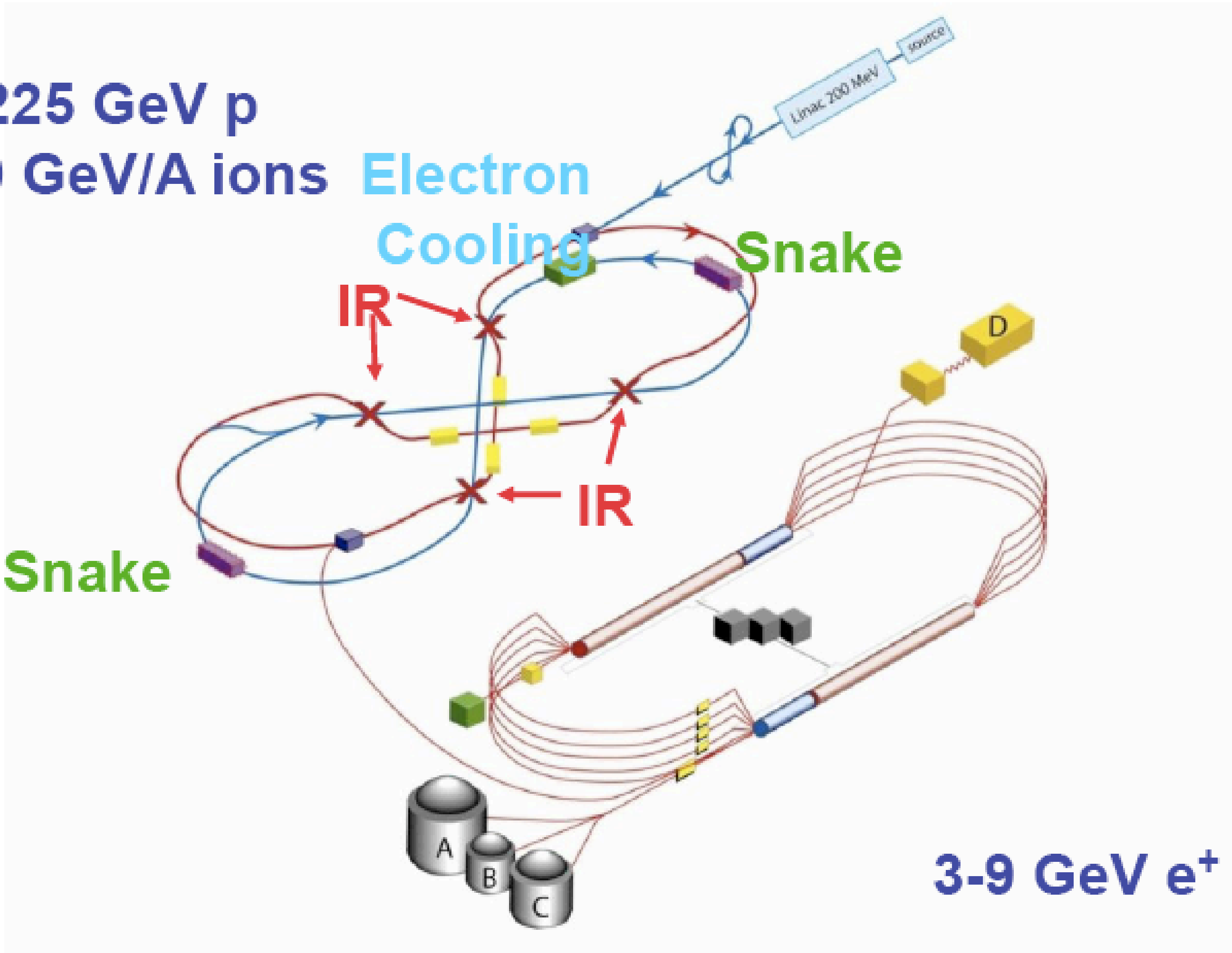
Based on sLHC. e^+ ? , w/o energy recovery

EIC JLAB proposal

Long-term Landscape : ELIC

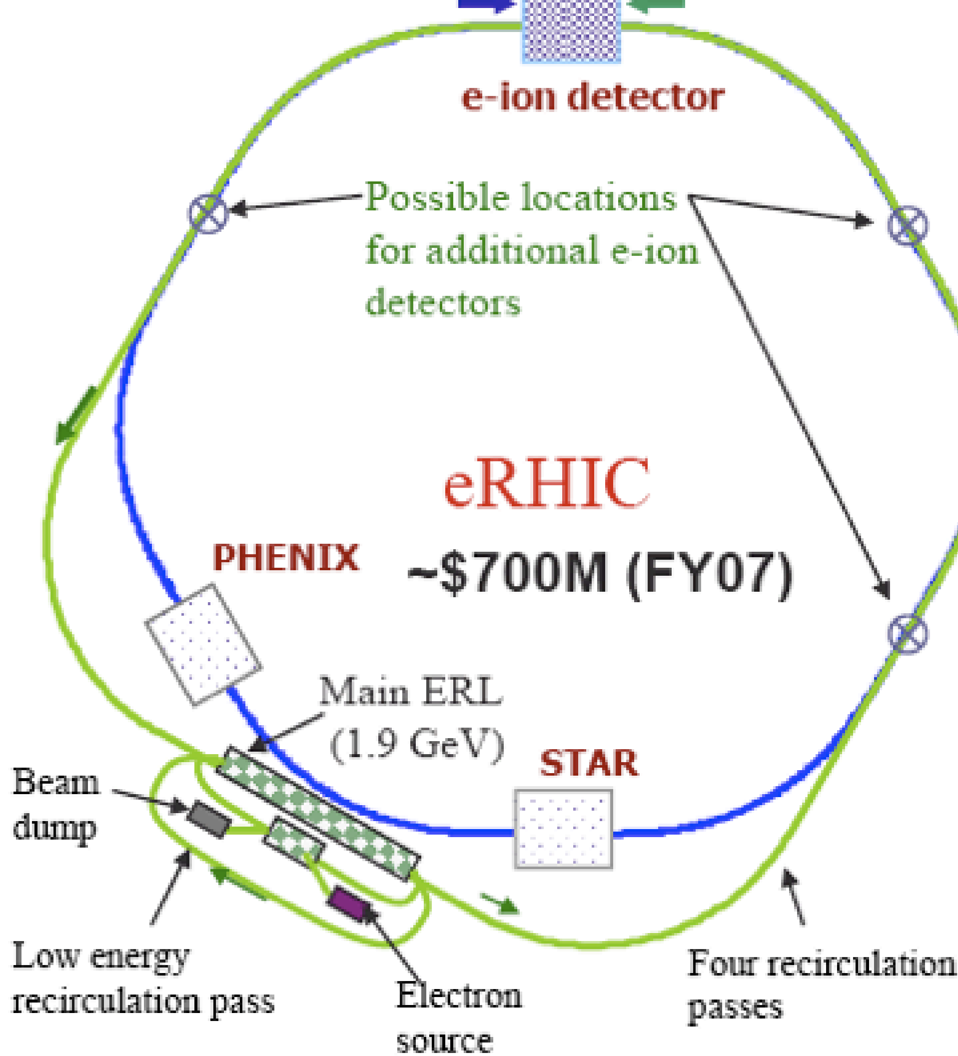
30-225 GeV p

30-100 GeV/A ions



3-9 GeV e^+ and e^-

Long-Term (>2020) Future of QCD Physics at RHIC: EIC → eRHIC



Add ERL injector with polarized e^- source to enable $\vec{e}+\vec{p}$, $^3\vec{He}$ and e+A (up to Uranium) to study matter in gluon-dominated regime

- 10 GeV electron design energy. Possible upgrade to 20 GeV by doubling main linac length.
- 5 recirculation passes (4 in RHIC tunnel)
- Multiple electron-hadron interaction points (IPs) permit multiple detectors;
- Full polarization transparency at all energies for the electron beam;
- Ability to take full advantage of transverse cooling of the hadron beams;
- Possible options to include polarized positrons at lower luminosity: compact storage ring or ILC-type $\vec{e}^+$ source
- R&D already under way on various accelerator issues; more to come.

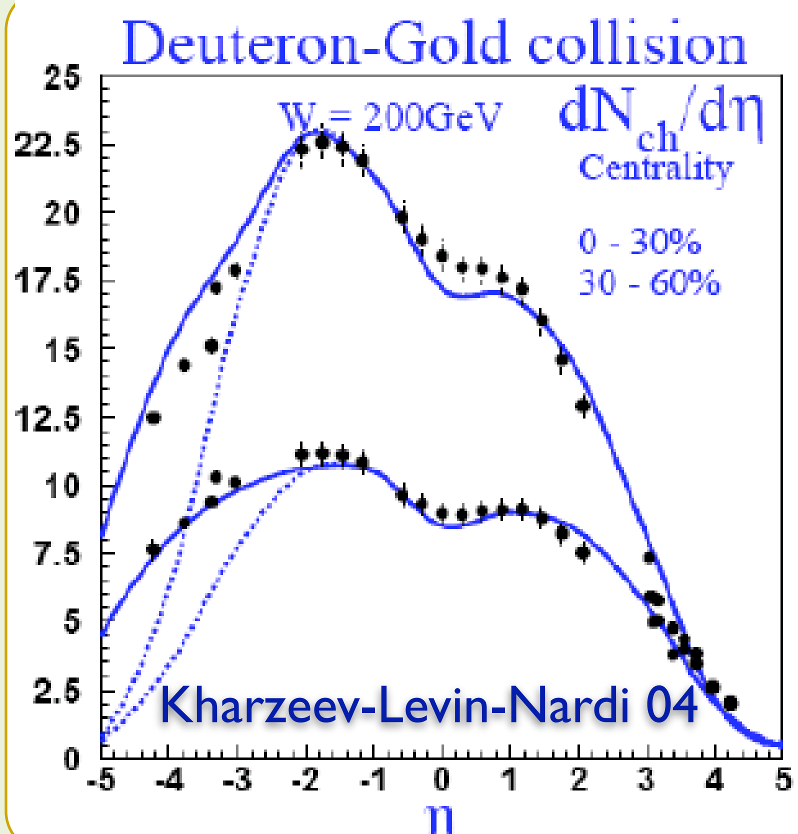
- ***Subsequent stages/ alternative layouts could increase e-beam & ion-beam energies and $\mathcal{L}$ from nominal 10 × 250 GeV, ~3 × 10^{33} cm^{-2}s^{-1} $\vec{e}+\vec{p}$***

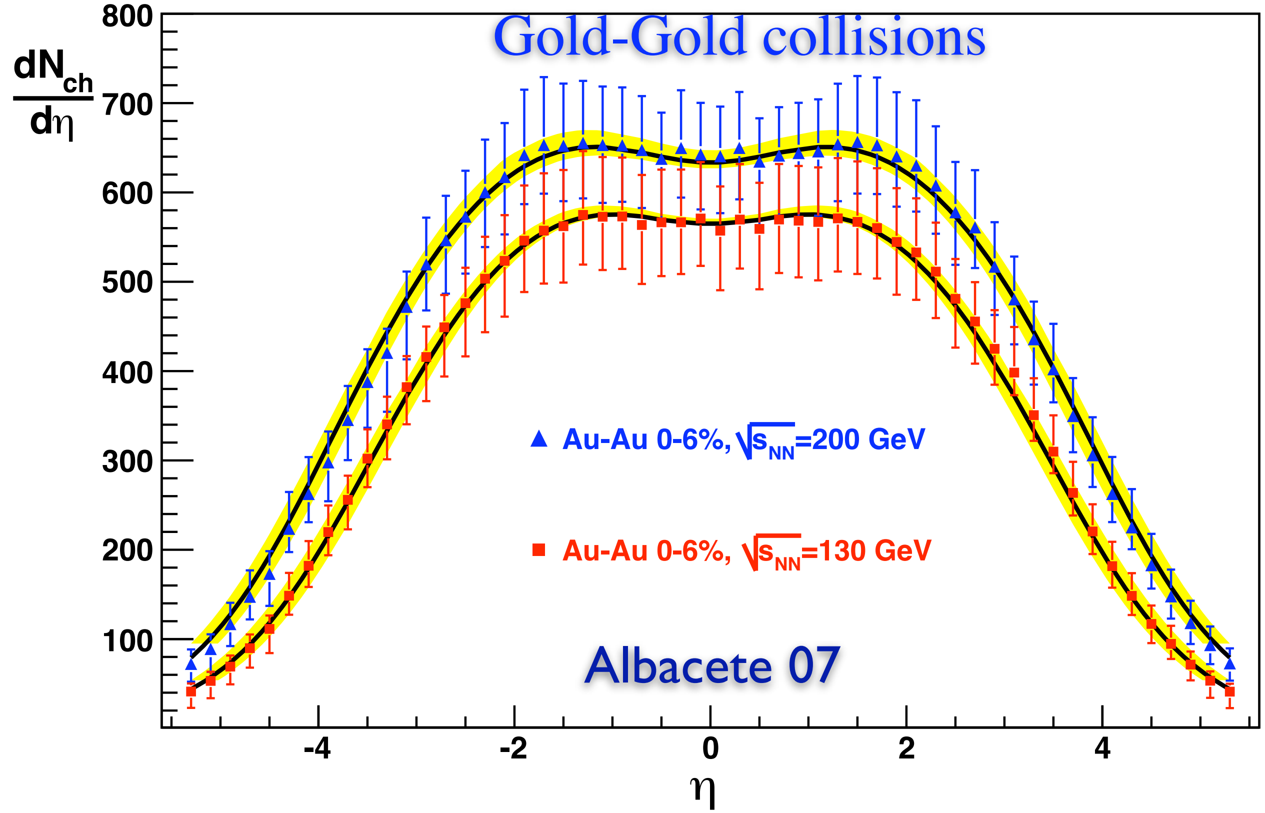
⇒ Saturation-based calculations describe the energy, rapidity and centrality dependence of multiparticle production at RHIC Au-Au and d-Au collisions

k_t-factorization + saturation + local parton-hadron duality

$$\frac{dN^g_{AB}}{d\eta} \sim \alpha_s \int \frac{d^2p}{p^2} \int d^2k \, \varphi_A(x_1, k) \, \varphi_B(x_2, |p-k|) \quad \text{with} \quad x_{1(2)} = \frac{p_t}{\sqrt{s}} e^{\pm\eta}$$

Multiplicity density

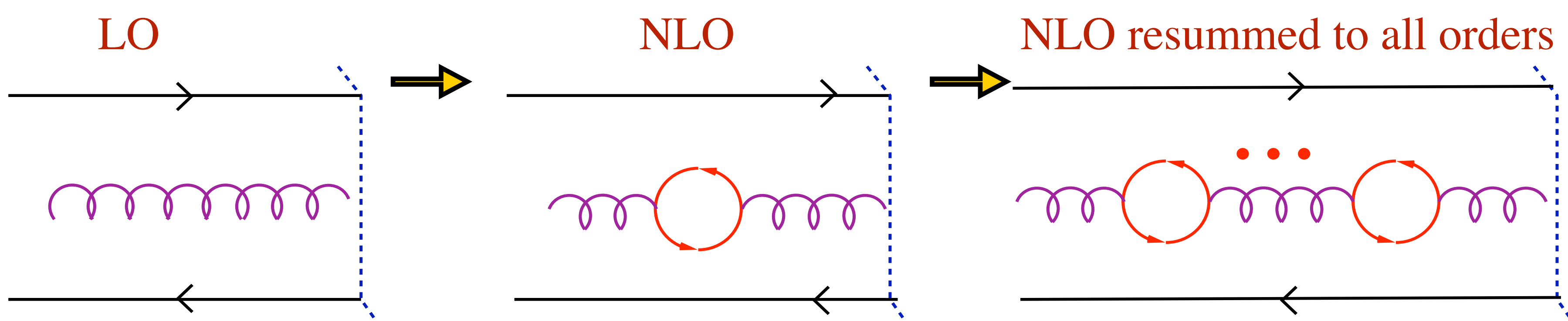




Results obtained using running coupling BK

- Running coupling corrections to BK-JIMWLK. Three independent calculations
  - ⇒ "Shock wave" method: I. Balitsky: hep-ph/0609115
  - ⇒ Light Cone Perturbation Theory: Y. Kovchegov and H. Weigert: hep-ph/0609090
  - ⇒ Dispersive methods and Borel resummation: E. Gardi et. al: hep-ph/0609087

- General strategy: All order resummation of $\alpha_s N_f$ contributions from quark loops:



$$\alpha_\mu \ln\left(\frac{1}{x}\right)\left[1+\sum_n c_n\left(\alpha_\mu \frac{N_f}{6\pi}\ln\frac{q^2}{\mu^2}\right)^n\right]$$

Resumming the geometric series $N_f \to -6\pi\beta_2$

$$\Rightarrow \ln(1/x)\,\frac{\alpha_\mu}{1+\alpha_\mu\beta_2\ln\left(\frac{C\,q^2}{\mu^2}\right)}$$

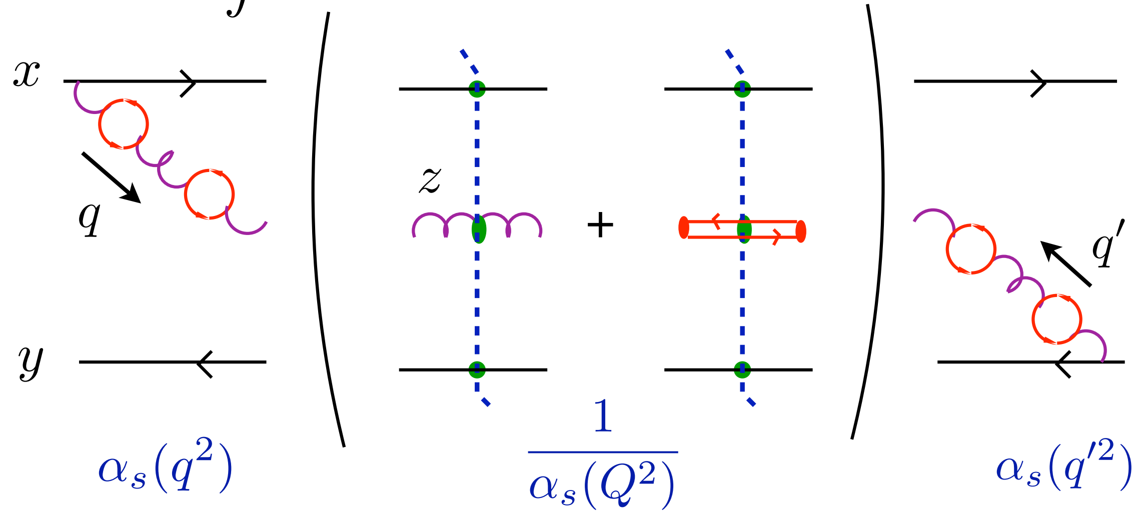
- Fourier transform to coordinate space (R). Brodsky-Lepage-Mackenzie scale setting:

$$\alpha_\mu \ln\left(\frac{1}{x}\right)\left[1-\beta_2\left(c_0+c_1\,\alpha_\mu\ln\left(\frac{4}{R^2\,\mu^2}\right)\right)+\dots\right] \Rightarrow \ln\left(\frac{1}{x}\right)\frac{\alpha_\mu}{1+\alpha_\mu\beta_2\ln\left(\frac{4\,e^{c_1}}{R^2\mu^2}\right)}$$

- Complete (all orders in $\alpha_s\beta_2$) evolution equation: $\Rightarrow$ $$\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$$

JLA and Y. Kovchegov PRD 75 125021 (07):

$\Rightarrow$ ***Running*** **term:** $\mathcal{R}[S] = \int d^2z\, \tilde{K}(\underline{r}, \underline{r}_1, \underline{r}_2)\, [S(\underline{x}, \underline{z}) S(\underline{z}, \underline{y}) - S(\underline{x}, \underline{y})]$



+ virtual terms

$$\tilde{K}_{KW}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c}{2\pi^2} \left[\frac{\alpha_s(r_1^2)}{r_1^2} - 2\frac{\alpha_s(r_1^2)\alpha_s(r_2^2)}{\alpha_s(R^2)} + \frac{\alpha_s(r_2^2)}{r_2^2} \right]$$

$$\tilde{K}_{Bal}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c\, \alpha_s(r^2)}{2\pi^2} \left[\frac{r^2}{r_1^2\, r_2^2} + \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

- The qq contribution ensures the renormalizability of the all orders in $\alpha_s\beta_2$ corrections and the right physical behavior of the running term:

$$\mathcal{R}[S] \to 0 \quad \text{for} \begin{cases} S \to 0 \\ S \to 1 \end{cases}$$

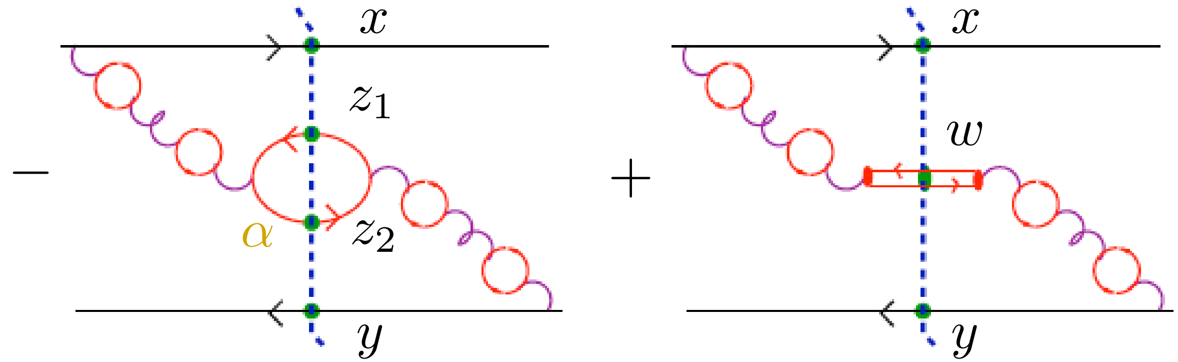
$\Rightarrow$ Probability conservation

$\Rightarrow$ Unitarity:

$$\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$$

⇒ ***Subtraction*** **term:**

$$\mathcal{S}[S] = \int d^2 z_1 d^2 z_2 \, K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2) \left[S(\underline{x}, \underline{w}) S(\underline{w}, \underline{y}) - S(\underline{x}, \underline{z}_1) S(\underline{z}_2, \underline{y}) \right]$$



$N_f \longrightarrow -6\pi\beta_2$

$$K_{sub}(\underline{x}, \underline{y}, \underline{z}_1, \underline{z}_2) = -\frac{3\beta_2}{2\pi^3} \int_0^1 d\alpha \, \frac{1}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2] \, [\alpha(\underline{z}_1 - \underline{y})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2] \, z_{12}^4}$$

$$\left\{ \left[-4\alpha\bar{\alpha} \, \underline{z}_{12} \cdot (\underline{z} - \underline{x}) \, \underline{z}_{12} \cdot (\underline{z} - \underline{y}) + z_{12}^2 (\underline{z} - \underline{x}) \cdot (\underline{z} - \underline{y}) \right] \alpha_s(R_T(\underline{x})) \, \alpha_s(R_T(\underline{y})) \right.$$

$$2\alpha\bar{\alpha}(\alpha - \bar{\alpha}) z_{12}^2 \left[\underline{z}_{12} \cdot (\underline{z} - \underline{x}) \, \alpha_s(R_T(\underline{x})) \, \alpha_s(R_L(\underline{y})) + \underline{z}_{12} \cdot (\underline{z} - \underline{y}) \, \alpha_s(R_L(\underline{x})) \, \alpha_s(R_T(\underline{y})) \right]$$

$$\left. 4\alpha^2 \bar{\alpha}^2 z_{12}^4 \, \alpha_s(R_L(\underline{x})) \, \alpha_s(R_L(\underline{y})) \right\}$$

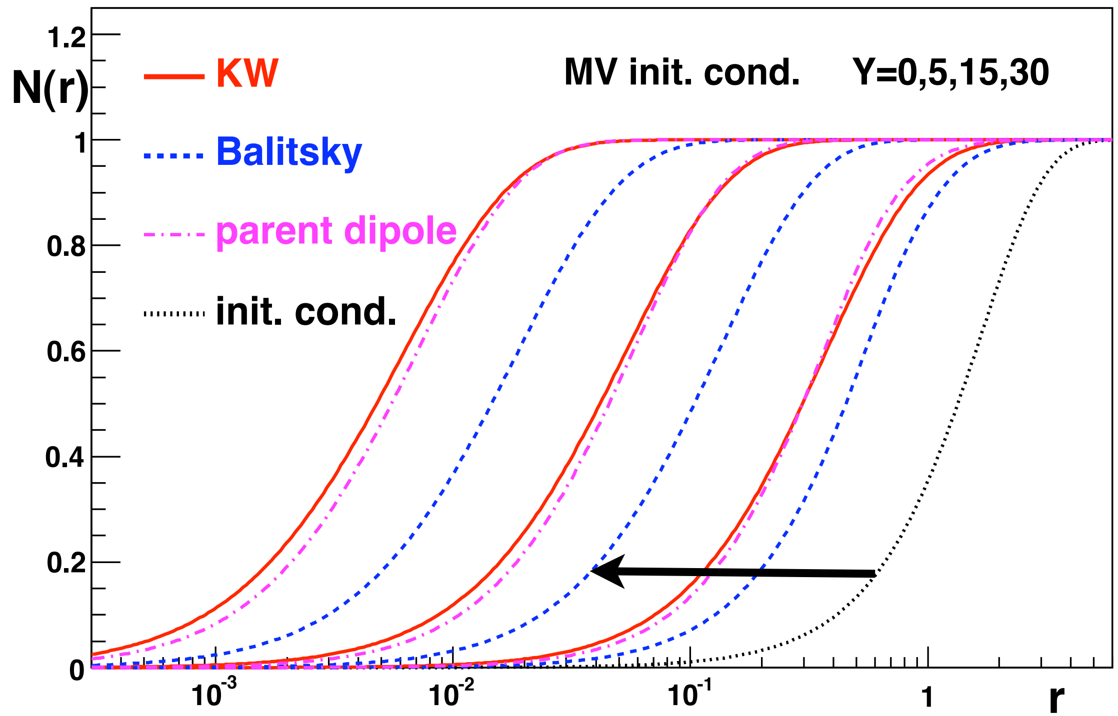
- It receives contributions from transverse (T) and longitudinal (L) gluon's polarization:

$$\ln\left(\frac{1}{R_T^2(\underline{x})\mu^2}\right) = \ln\left(\frac{4e^{-2\gamma-5/3}}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2]\mu^2}\right) + \frac{\alpha\bar{\alpha}\, z_{12}^2}{(\underline{z} - \underline{x})^2} \ln\left(\frac{\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2}{\alpha\bar{\alpha} z_{12}^2}\right)$$

$$\ln\left(\frac{1}{R_L^2(\underline{x})\mu^2}\right) = \ln\left(\frac{4e^{-2\gamma-5/3}\, \alpha\bar{\alpha}\, z_{12}^2}{[\alpha(\underline{z}_1 - \underline{x})^2 + \bar{\alpha}(\underline{z}_2 - \underline{x})^2]^2 \mu^2}\right)$$

• The solutions corresponding to different prescriptions for the running coupling kernel differ considerably:

$$\frac{\partial S}{\partial Y} = \mathcal{R}\,[S]$$



• Solutions with KW prescription lie pretty close to those obtained with ***parent dipole*** running:

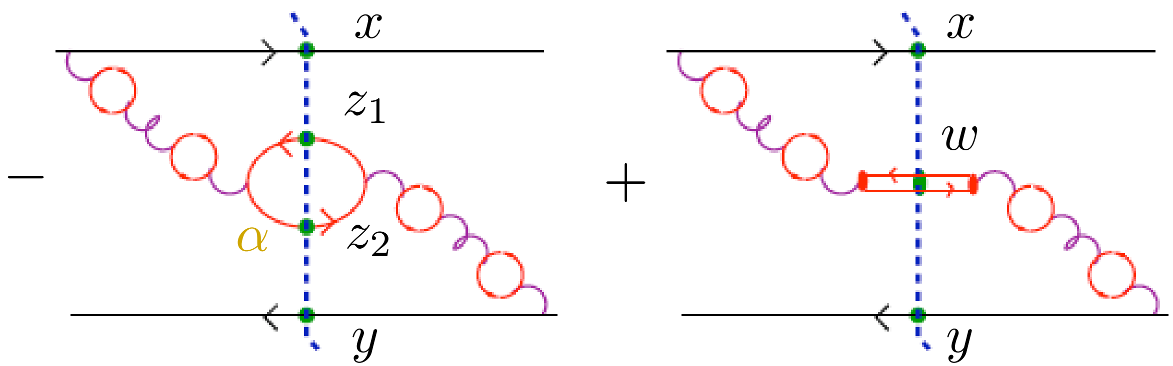
$$\tilde{K}(\underline{r}, \underline{r}_1, \underline{r}_2) = \frac{N_c\, \alpha_s(r^2)}{2\pi^2}\, \frac{r^2}{r_1^2\, r_2^2}$$

• Large scheme dependence: Contrary to expectations, the subtraction contribution has to be large for the two calculations to agree

$$\frac{\partial S}{\partial Y} = \mathcal{R}[S] - \mathcal{S}[S]$$

⇒ ***Subtraction*** **term:**

$$\mathcal{S}[S] = \int d^2z_1 d^2z_2\, K_{sub}(\underline{x},\underline{y},\underline{z}_1,\underline{z}_2)\,\left[S(\underline{x},\underline{w})S(\underline{w},\underline{y}) - S(\underline{x},\underline{z}_1)S(\underline{z}_2,\underline{y})\right]$$



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$$\Big\{\left[-4\alpha\bar\alpha\,\underline{z}_{12}\cdot(\underline{z}-\underline{x})\,\underline{z}_{12}\cdot(\underline{z}-\underline{y}) + z_{12}^2(\underline{z}-\underline{x})\cdot(\underline{z}-\underline{y})\right]\,\alpha_s(R_T(\underline{x}))\,\alpha_s(R_T(\underline{y}))$$
$$2\alpha\bar\alpha(\alpha-\bar\alpha)z_{12}^2\left[\underline{z}_{12}\cdot(\underline{z}-\underline{x})\,\alpha_s(R_T(\underline{x}))\,\alpha_s(R_L(\underline{y})) + \underline{z}_{12}\cdot(\underline{z}-\underline{y})\,\alpha_s(R_L(\underline{x}))\,\alpha_s(R_T(\underline{y}))\right]$$
$$4\alpha^2\bar\alpha^2 z_{12}^4\,\alpha_s(R_L(\underline{x}))\,\alpha_s(R_L(\underline{y}))\Big\}$$

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$$\ln\left(\frac{1}{R_L^2(\underline{x})\mu^2}\right) = \ln\left(\frac{4e^{-2\gamma-5/3}\,\alpha\bar\alpha\, z_{12}^2}{[\alpha(\underline{z}_1-\underline{x})^2+\bar\alpha(\underline{z}_2-\underline{x})^2]^2\mu^2}\right)$$

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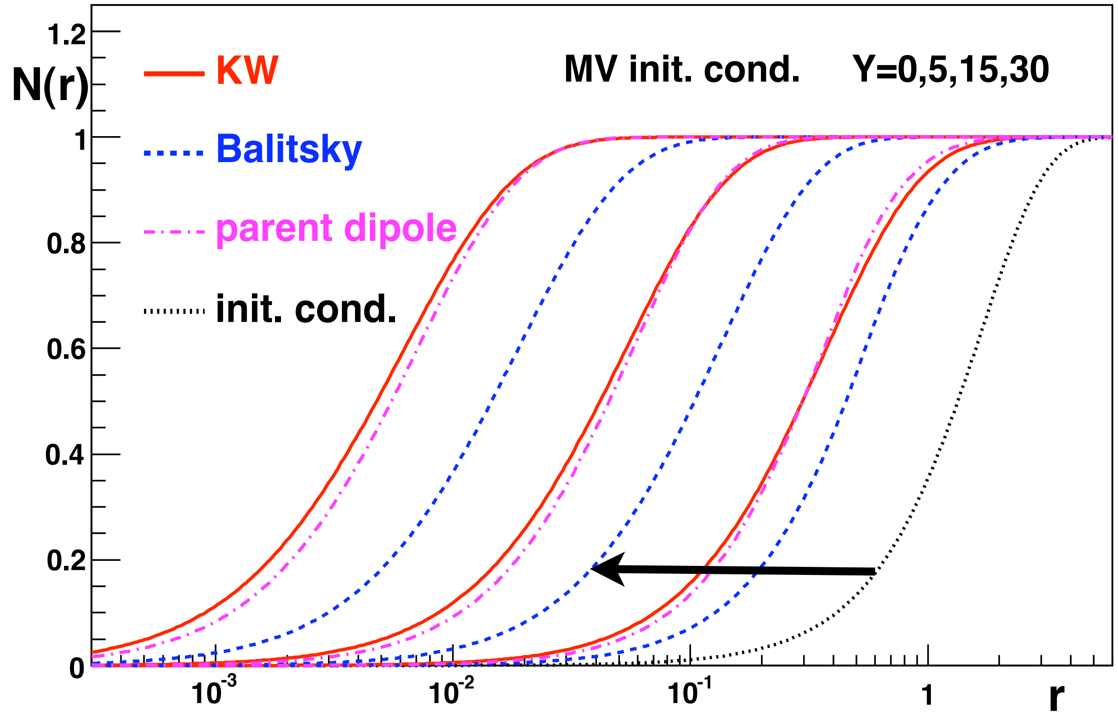

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• Large scheme dependence: Contrary to expectations, the subtraction contribution has to be large for the two calculations to agree

@ Particle production in A-A collisions (JLA arXiv.0707.2545 [hep-ph])

• k_t-factorization 'a la Kharzeev-Levin-Nardi'

$$\frac{dN_{AA}}{d\eta} \propto \frac{4\pi N_c}{N_c^2-1} \int^{p_m} \frac{d^2 p_t}{p_t^2} \int^{p} d^2 k_t \, \alpha_s(Q) \, \varphi_A\left(x_1; \frac{|p_t + k_t|}{2}\right) \varphi_A\left(x_2; \frac{|p_t - k_t|}{2}\right)$$

• 2→1 kinematics

$$x_{1(2)} = \frac{p_t}{\sqrt{s}} \, e^{\pm y}$$

or

$$x_{1(2)} = \frac{m_t}{\sqrt{s}} \, e^{\pm y}$$

• rapidity ↔ pseudorapidity: average hadron mass

$$y(\eta, p_t, m) = \frac{1}{2} \ln \left[\frac{\sqrt{\frac{m^2+p_t^2}{p_t^2} + \sinh^2\eta} + \sinh\eta}{\sqrt{\frac{m^2+p_t^2}{p_t^2} + \sinh^2\eta} - \sinh\eta} \right]$$

• Running coupling: $Q = \max\left\{\frac{|p_t \pm k_t|}{2}\right\}$

+

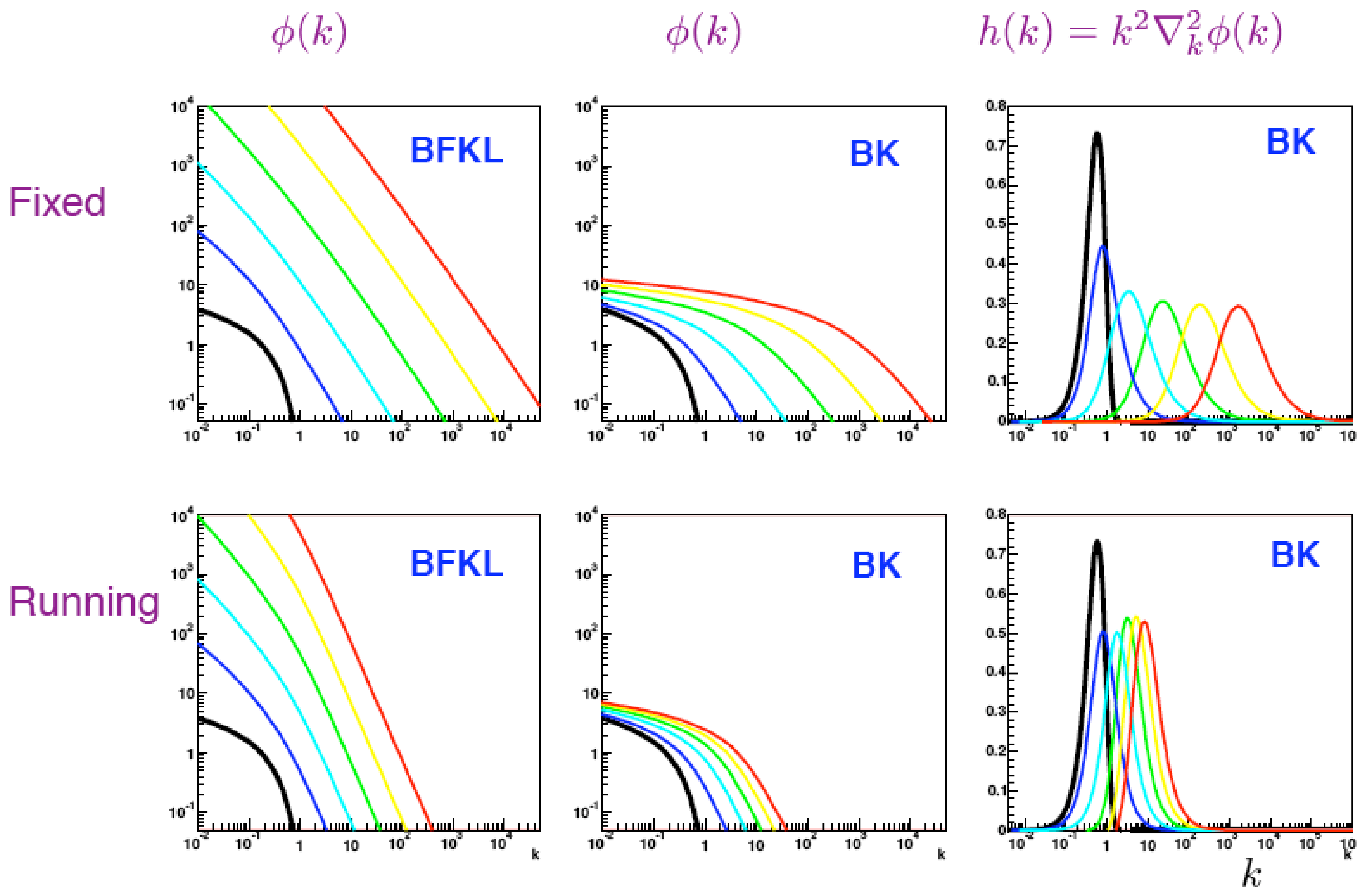
$$\varphi(x, k) = \int \frac{d^2 r}{2\pi^2 \, r^2} \exp\left[i \, \underline{k} \cdot \underline{r}\right] \mathcal{N}(Y, r)$$ Solutions of BK equation including all orders in $\alpha_s\beta_2$ corrections $\times (1-x)^4$

with $Y = \ln\left(\frac{0.05}{x}\right) + \Delta Y_{ev}$

+

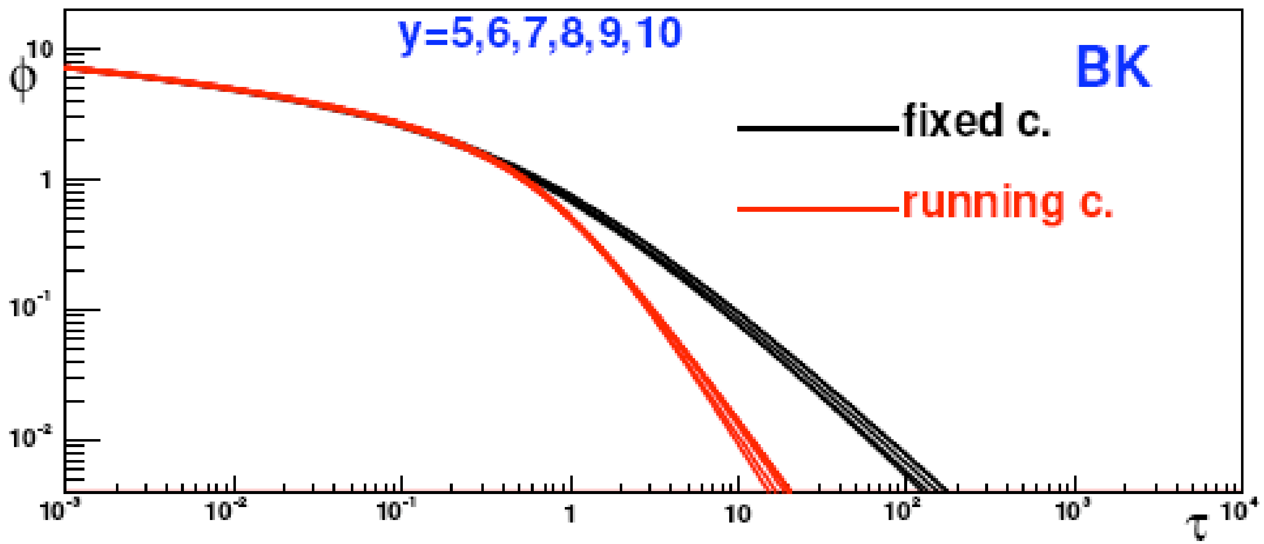
Local Hadron-Parton Duality

Numerical Solutions



- The solutions of the evolution at large rapidity exhibit the property of geometric scaling:

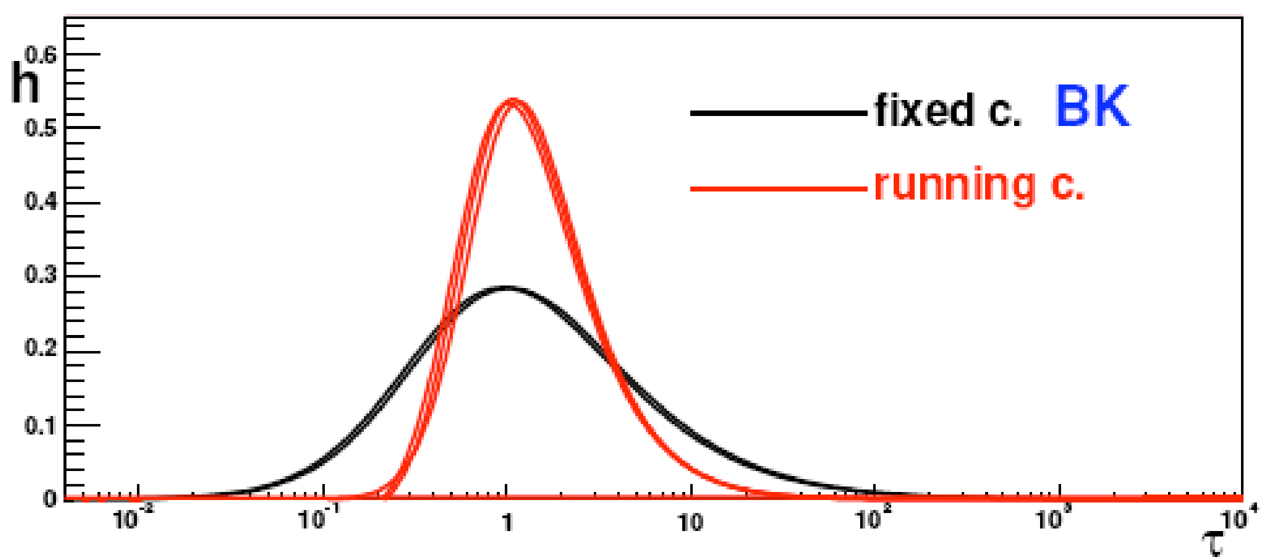
$$\phi(k,Y) \to \phi(\tau), \quad \tau = \frac{k}{Q_s(Y)}$$



- However, the scaling functions are different for fixed and running coupling

Fix: $\phi(\tau) \sim \tau^{-2(1-\lambda)} \ln(b\tau)$

Run: $\phi(\tau) \sim \tau^{-2(1-\lambda)} \ln(b\tau)$

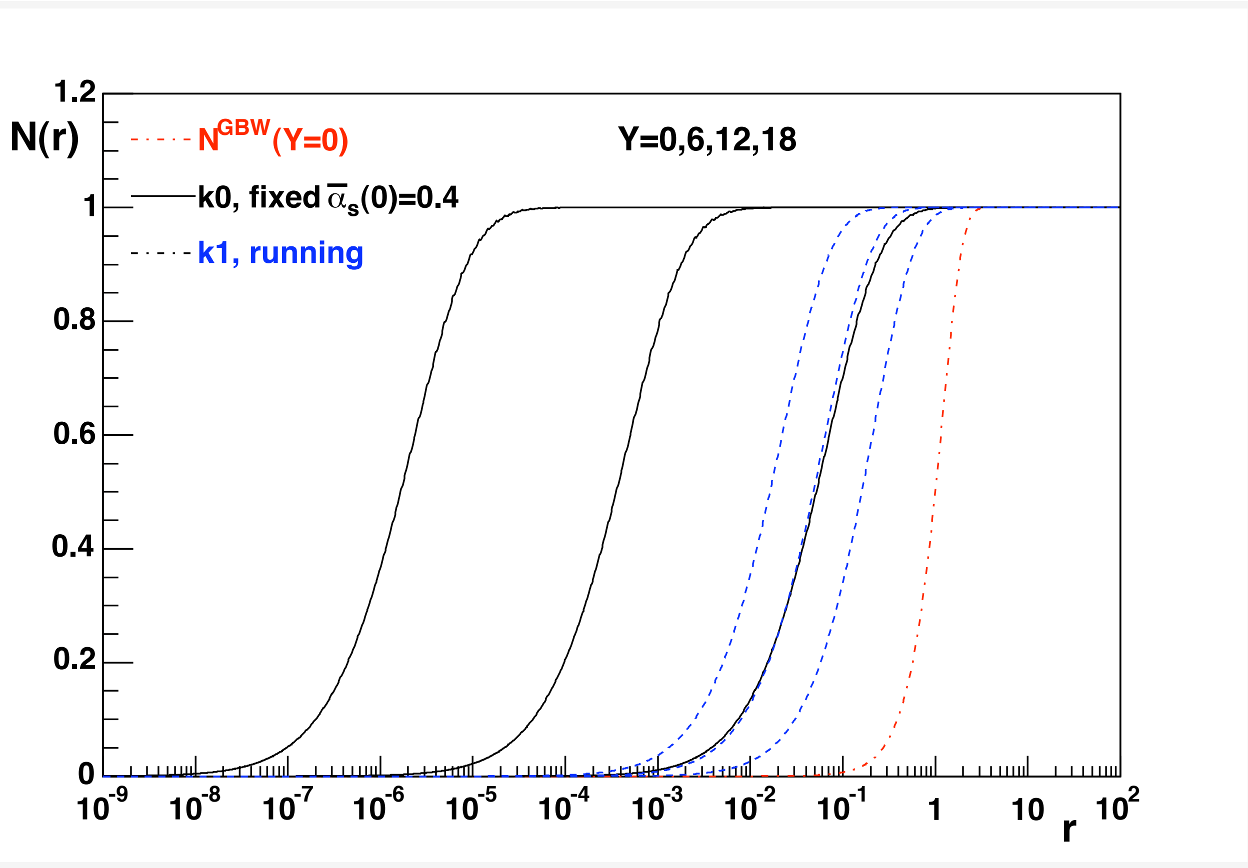


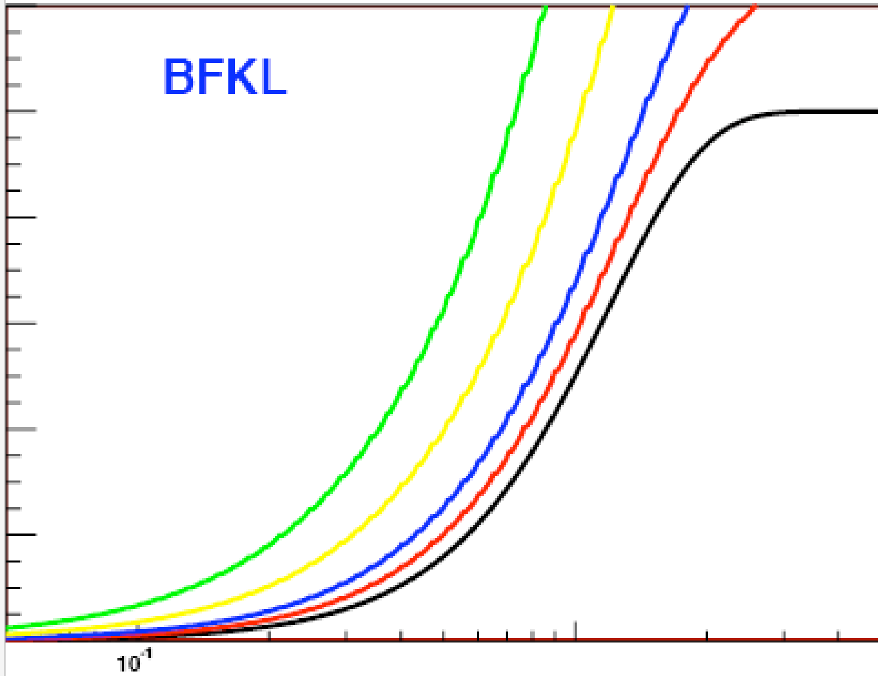
$$\lambda_{fix} \sim 0.37$$

$$\lambda_{run} \sim 0.15$$

$$\tau > 1$$

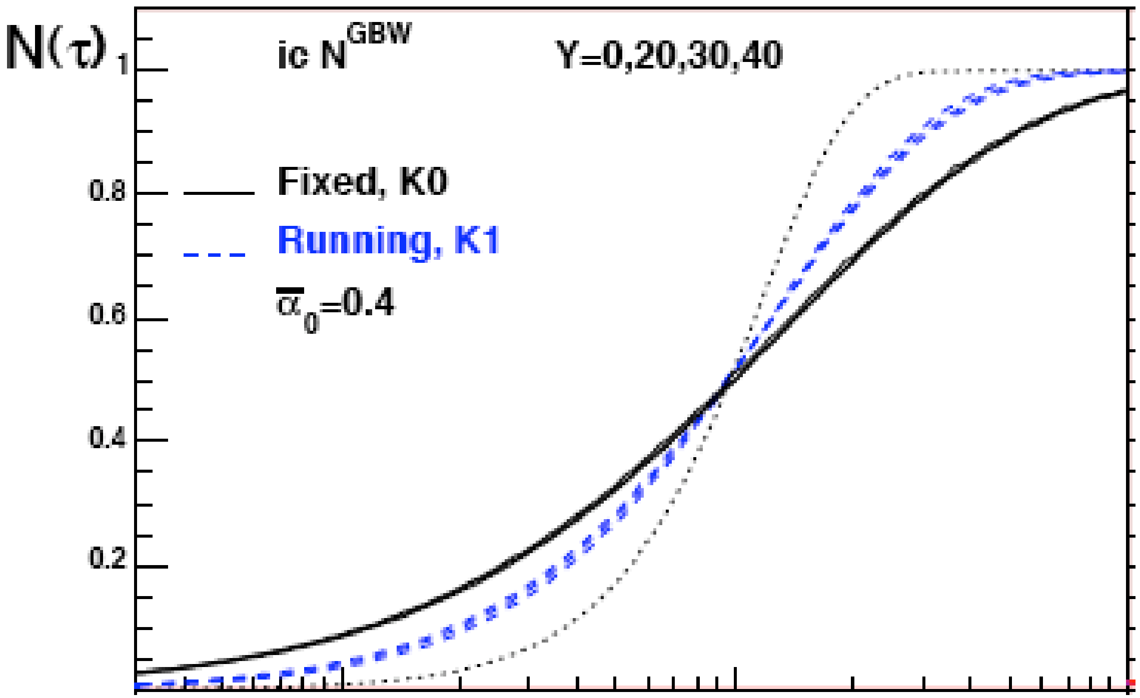
Numerical Solutions





- BFKL evolution clearly violates unitarity: N>1
- Running coupling effects considerably slow down the evolution w.r.t. the fixed coupling case (emission of small dipoles is suppressed)

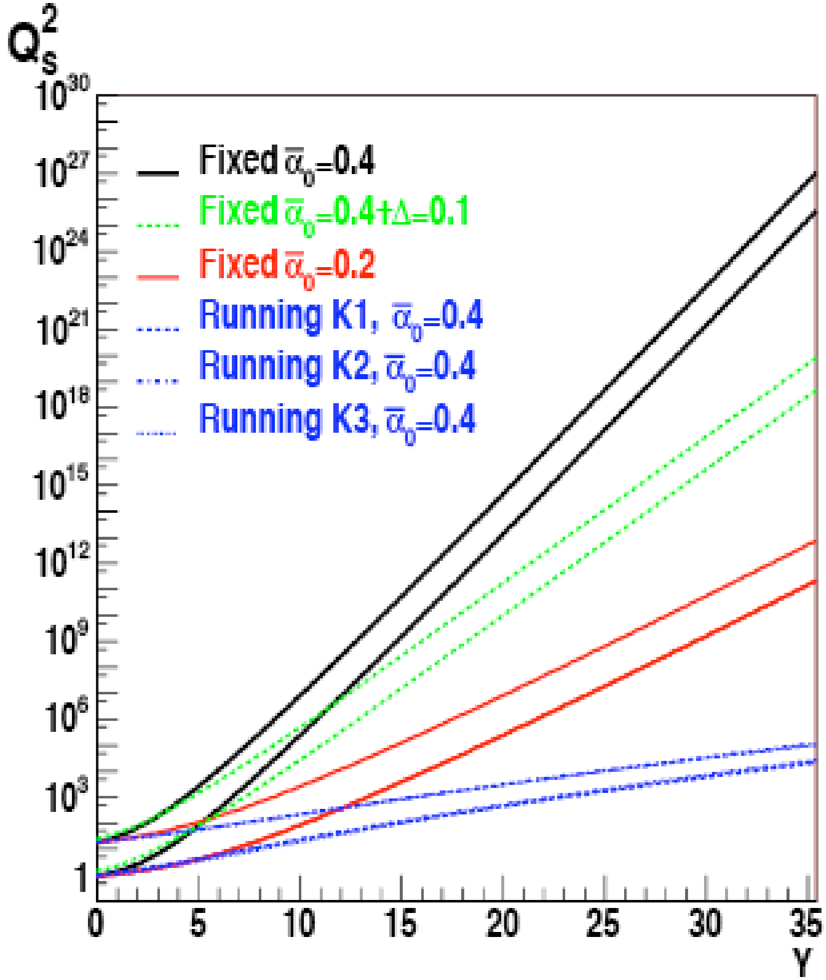
- Geometric scaling: $\mathcal{N}(Y,r) \to \mathcal{N}(\tau = r\,Q_s(Y))$



- Scaling fully realized at extremely large rapidities: Y~80.

- Fixed and running coupling scaling solutions are different.

- $Q_s^2(Y)$: $\mathcal{N}(Y, 1/Q_s) = 0.5$



$$Q^2_{fix,s}(Y) \sim \exp\left\{4.8\alpha_s Y\right\}$$

$$Q^2_{run,s}(Y) \sim \exp\left\{\sqrt{Y}\right\} \sim \exp\left\{0.3Y\right\}$$